Simulations of supersymmetric Yang-Mills theory on the lattice

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1. Supersymmetric Yang-Mills theory on the lattice
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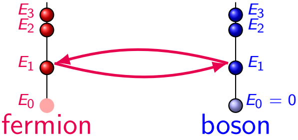
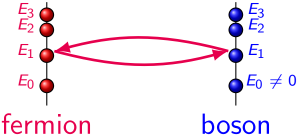
Supersymmetry

- fermions $\overset{\text{SUSY}}{\rightleftharpoons}$ bosons; $\left\{Q\,,\,\bar{Q}\right\} \sim \gamma^{\mu} P_{\mu}$

$\Rightarrow$ strict paring of states; except ground state





Δ can be $\neq 0$
unbroken SUSY

$\Delta = 0$; all states paired
spontaneous SUSY breaking

- Witten index[1]:
  $\Delta = n_B^{E=0} - n_F^{E=0} = \mathrm{Tr}(-1)^F = \lim_{\beta\to 0} \mathrm{Tr}(-1)^F \exp(-\beta H)$

[1] [Witten, Nucl.Phys.B202 (1982)]

Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\psi}\not{D}\psi - \frac{m_g}{2}\bar{\psi}\psi\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- ψ Majorana fermion in the adjoint representation
- gluino mass term $m_g \Rightarrow$ soft SUSY breaking

Lattice supersymmetry

- contradiction: locality $\not\leftrightarrow$ lattice SUSY[1]
- no Ginsparg-Wilson solution (so far)[2]

$\Rightarrow$ fine tuning problem

- low dimensions: fine tuning/locality problem solved[3]
- SYM theory: tuning possible

[1] [Kato, Sakamoto, So, JHEP 0805 (2008)], [GB, JHEP 1001 (2010)]

[2] [GB, Bruckmann, Pawlowski, Phys. Rev. D 79 (2009)]

[3] [Golterman,Petcher Nucl. Phys. B319 (1989)],
[Catterall, Gregory, Phys. Lett. B 487 (2000)],
[Giedt, Koniuk, Poppitz, Yavin, JHEP 0412 (2004)],
[G.B, Kästner, Uhlmann, Wipf, Annals Phys. 323 (2008)],
[Baumgartner, Wenger, PoS LATTICE 2011],. . .

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- "brute force" discretization: Wilson fermions

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right]$$

$$T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu}); \quad \kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$
- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

Recovering symmetry

Ward identities of supersymmetry and chiral symmetry:

- tuning of $\kappa(m_g)$ to recover chiral symmetry [1]
- same tuning to recover supersymmetry [2]

Fine-tuning:

chiral limit = SUSY limit $+O(a)$, obtained at critical κ

- good realization: overlap/domainwall fermions (but too expensive)[3]

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

The sign problem in supersymmetric theories

- $Z \propto \Delta$ (periodic boundary conditions)
- $\Delta = 0$ from fluctuating sign of fermion path integral
- Majorana fermions:

$$\int \mathcal{D}\psi e^{-\frac{1}{2}\int \bar{\psi} D\psi} = \mathrm{Pf}(CD) = \mathrm{sign}(\mathrm{Pf}(CD))\sqrt{\det D}$$

$\Rightarrow$ severe sign problem if spontaneous SUSY breaking possible[1]

[1] [Wozar, Wipf, Annals Phys. 327 (2012)], [Wenger]

The Sign problem in SYM and on the lattice

- continuum $SU(N)$ supersymmetric Yang-Mills theory: $\Delta = N$

$\Rightarrow$ no sign problem in the continuum

- Wilson fermions: sign problem even in SYM
- reweighting: $\mathrm{sign}(\mathrm{Pf}(CD))$
- general lattice SUSY: modification of fermion path integral by Wilson term requires special concern

Sign problem and eigenvalues

- γ_5-Hermiticity: $\gamma_5 \, \mathrm{D} \, \gamma_5 = D^\dagger \Rightarrow$ pairing λ, λ^*
- charge conjugation: $C \, \mathrm{D} \, C^T = \mathrm{D}^T$
  $\Rightarrow$ degenerate eigenvalues $\lambda_1 = \lambda_{N/2+1}$

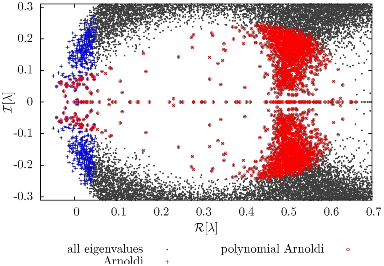
$\Rightarrow$ $\det(\mathrm{D}) = \prod^{N/2} \lambda_i^2$ positive

- $|\,\mathrm{Pf}(C(\mathrm{D} - \sigma \mathbb{1}))| = \sqrt{\det(\mathrm{D} - \sigma \mathbb{1})} = \prod_{i=1}^{N/2} |\lambda_i - \sigma|$
- Pfaffian polynomial in σ

$$\Rightarrow \qquad \mathrm{Pf}(C\,\mathrm{D}) = \prod_{i=1}^{N/2} \lambda_i$$

- number of negative paired real eigenvalues of D even / odd
  $\Rightarrow$ positive / negative Pfaffian

Sign problem and the eigenvalues



- contribution of neg. signs: reduced in continuum limit; enlarged in chiral limit
- methods: next talk
- further applications: determinant sign in $N_f = 1$ QCD
- further applications: spectral decomposition, index

Status of the simulations

- main focus: mass-spectrum of SYM
- simulations similar to $N_f = 1$ QCD
- PHMC: approximate $|\,\mathrm{Pf}(CD)|$
- improvements: tree level Symanzik improved gauge action; stout smearing
- lightest particles hard to measure: mesons with disconnected contributions; glueballs
- improvements: spectral decomposition, smearing techniques

Low energy effective theory

confinement like in QCD $\Rightarrow$ colorless low energy bound states

multiplet[1]:
mesons : $a-f_0$: $\bar{\lambda}\lambda$; $a-\eta'$: $\bar{\lambda}\gamma_5\lambda$
fermionic gluino-glue ($\sigma_{\mu\nu}F_{\mu\nu}\lambda$)

multiplet[2]:
glueballs: 0^{++}, 0^{-+}
fermionic gluino-glue

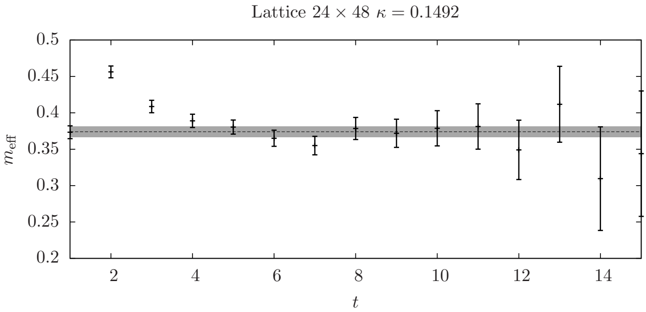
Supersymmetry

All particles of a multiplet must have the same mass (scalar, pseudoscalar, fermion).

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]
[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

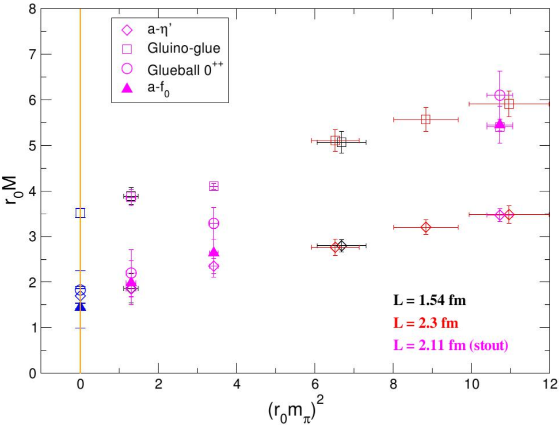
The gluino-glue particle

- gluino-glue fermionic operator $\sigma^{\mu\nu}\mathrm{Tr}[F_{\mu\nu}\lambda]$
- $F_{\mu\nu}$ represented by clover plaquette



$\Rightarrow$ APE smearing on gauge fields + Jacobi smearing on λ

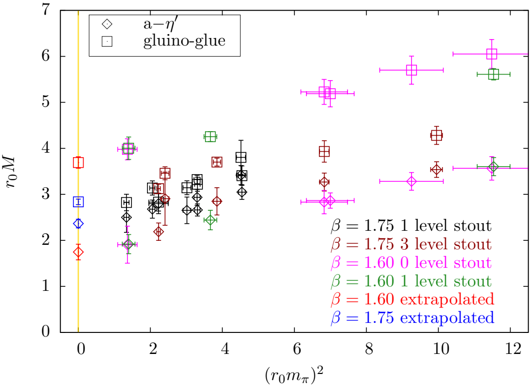
Mass gap at $\beta = 1.6$ [1]



⇒ unexpected mass gap

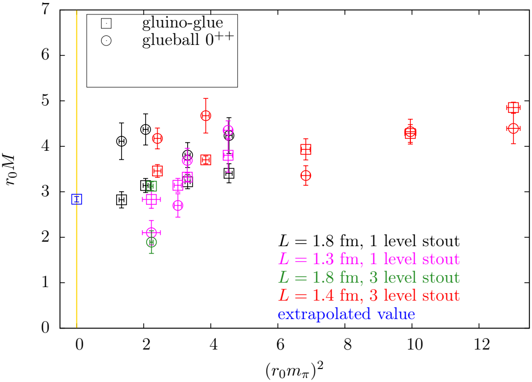
[1] [Demmouche et al., Eur.Phys.J.C69 (2010)]

The influence of the finite lattice spacing



⇒ smaller lattice spacing considerably reduces the mass gap

The results of the mass spectrum: $L = 1.35$fm



- still difficult to determine glueballs and $a - f_0$
- masses of the multiplet close to each other

Conclusions and outlook

- mass gap might be due to lattice artifacts
- finite size effects: increase mass gap, but negligible in current simulations
- mass splitting is already hard to measure at $\beta = 1.75$ on a $24^3 \times 48$ lattice
- most important limitation: need large statistic, especially for the scalar particles (0^{++}, $a - f_0$)

$\Rightarrow$ further improvements are investigated extended stout, clover