The gluino-glue particle and relevant scales for the simulations of supersymmetric Yang-Mills theory

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Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\psi}\not{D}\psi - \frac{m_g}{2}\bar{\psi}\psi\right]$$

- supersymmetry transformations:
  bosonic field $F_{\mu\nu}$ (gluon) $\leftrightarrows$ fermionic field ψ (gluino)
- ψ Majorana fermion in the adjoint representation
- gluino mass term $m_g \Rightarrow$ soft SUSY breaking

Motivation to study SYM theory on the lattice

1. Supersymmetric extension of the standard model
   - gauge part of SUSY models: super Yang-Mills theory (SYM)
   - non-perturbative sector important: breaking mechanism etc.
2. Connection to QCD
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD
3. Test lattice methods
   - Can SUSY be realized on the lattice ?
   - adjoint - 1/2 - flavor - QCD

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Low energy effective theory

confinement like in QCD $\Rightarrow$ colorless low energy bound states

multiplet[1]:
mesons : $a-f_0$: $\bar{\lambda}\lambda$; $a-\eta'$: $\bar{\lambda}\gamma_5\lambda$
fermionic gluino-glue ($\sigma_{\mu\nu}F_{\mu\nu}\lambda$)

multiplet[2]:
glueballs: 0^{++}, 0^{-+}
fermionic gluino-glue

Supersymmetry

All particles of a multiplet must have the same mass (scalar, pseudoscalar, fermion).

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]
[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- "brute force" discretization: Wilson fermions

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right]$$

$$T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu}); \quad \kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$
- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

Recovering symmetry

Ward identities of supersymmetry and chiral symmetry:

- tuning of $\kappa(m_g)$ to recover chiral symmetry [1]
- same tuning to recover supersymmetry [2]

Fine-tuning:

chiral limit = SUSY limit $+O(a)$, obtained at critical κ

determination of critical κ:

- chiral Ward identities: contain anomaly
- possible tuning: supersymmetric Ward identities
- better signal: mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

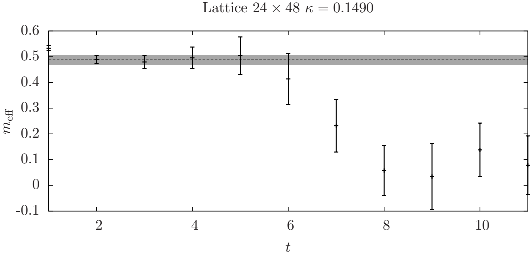
Details of the simulations

- path integral of Majorana fermions: $\mathrm{Pf}(C\,\mathrm{D}) = \mathrm{sign}(\mathrm{Pf}(C\,\mathrm{D}))\sqrt{\det(C\,\mathrm{D})}$
- PHMC: $\sqrt{\det(\mathrm{D})}$
- reweighting: $\mathrm{sign}(\mathrm{Pf}(C\,\mathrm{D}))$
- reweighting factors from negative real eigenvalues of D
- improvements: tree level Symanzik improved gauge action; stout smearing

The spectrum of SYM on the lattice: bosonic operators

- glueball operators

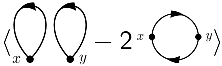
0^{++}



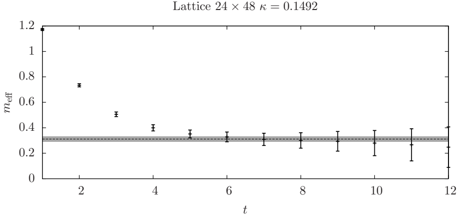
(0^{-+} currently insufficient statistic to obtain masses)

$\Rightarrow$ variational smearing methods (APE, HYP)

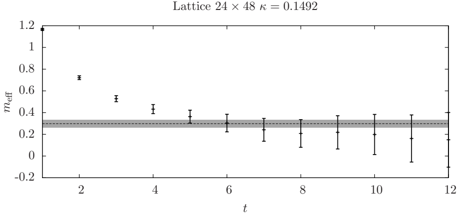
The spectrum of SYM on the lattice: bosonic operators

- mesons $\langle \bar{\lambda}(x)\gamma_5\lambda(x)\,\bar{\lambda}(y)\gamma_5\lambda(y)\rangle = \langle$  $\rangle$

pseudoscalar $a-\eta'$



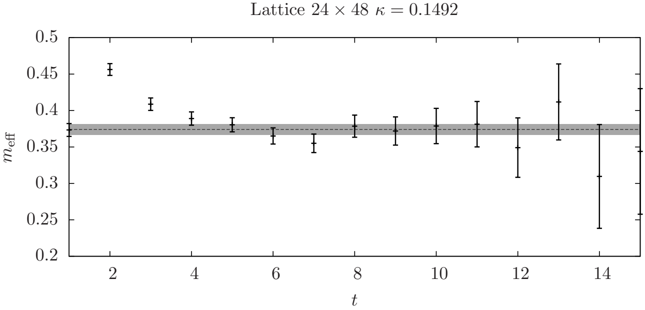
scalar $a - f_0$



$\Rightarrow$ disconnected contributions: SET + TEA/precond. TEA + TS

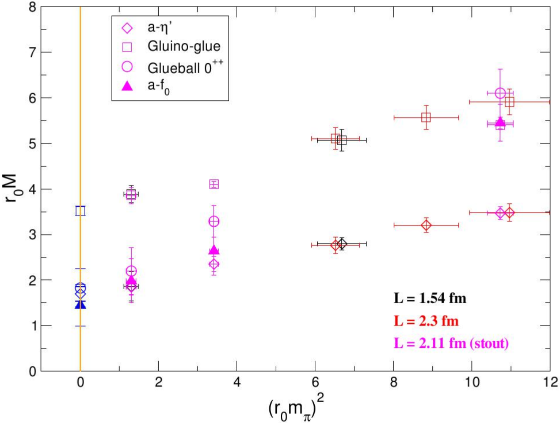
The gluino-glue particle

- gluino-glue fermionic operator $\sigma^{\mu\nu}\mathrm{Tr}[F_{\mu\nu}\lambda]$
- $F_{\mu\nu}$ represented by clover plaquette



$\Rightarrow$ APE smearing on gauge fields + Jacobi smearing on λ

Mass gap at $\beta = 1.6$ [1]



$\Rightarrow$ estimate finite size effects and lattice artifacts

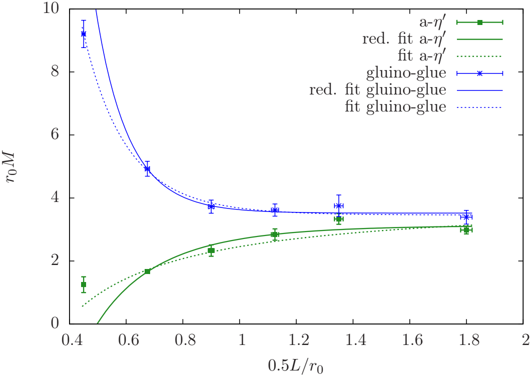
[1] [Demmouche et al., Eur.Phys.J.C69 (2010)]

Estimating finite size effects

- asymptotic behavior[1] for large L:
  $m(L) \approx m_0 + CL^{-1} \exp(-\alpha m_0 L)$
- best signal: gluino-glue
- reasonable signal: $a - \eta'$, but large deviation from asymptotic behavior due to systematic errors (excited states, disconnected contributions)
- simulations at lattice sizes:
  $8^3 \times 16$, $12^3 \times 24$, $16^3 \times 36$, $20^3 \times 40$, $24^3 \times 48$, $32^3 \times 64$
- chiral extrapolation of infinite volume limit at different $m_{a-\pi}$

[1] [Lüscher, Commun. Math. Phys. 104 (1986)], [Münster, Nucl. Phys. B 249 (1985)]

Dependence of the mass gap on the finite volume[1]

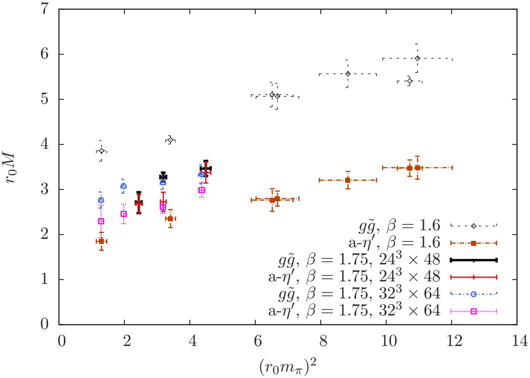


⇒ finite volume effects increase mass gap

⇒ influence of finite size effects small at moderate lattice sizes

[1] [GB, Berheide, Montvay, Münster, Özugurel, Sandbrink, arXiv:1206.2341]

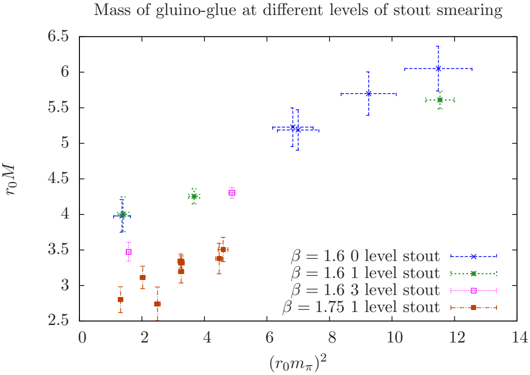
The influence of the finite lattice spacing



$\Rightarrow$ smaller lattice spacing considerably reduces the mass gap

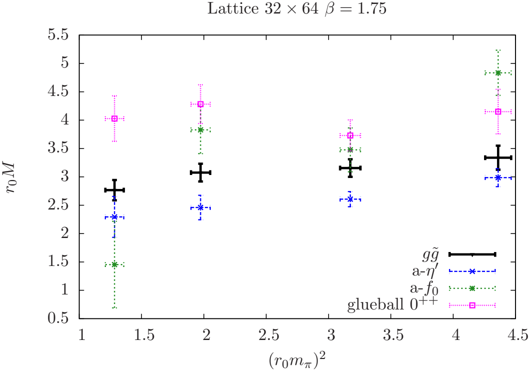
$\Rightarrow$ $24^3 \times 48$ ($L = 1.35$fm) consistent with $32^3 \times 64$ ($L = 1.8$fm)

Improvements of the action



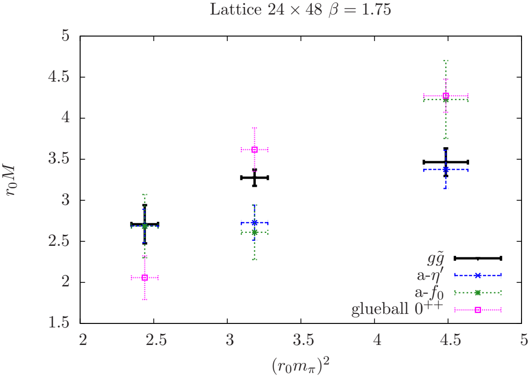
- indicates a reduction of the mass gap due to the stout smearing

The results of the mass spectrum: $L = 1.8$fm



- reasonable signal for gluino-glue and $a - \eta'$
- no conclusive results for the glueballs and $a - f_0$

The results of the mass spectrum: $L = 1.35$fm



- larger statistic at smaller lattice size
- better signal for glueballs and $a - f_0$
- masses of the multiplet close to each other

Conclusions and outlook

effects that increase mass gap	**relevance in current simulations**
finite size effects	negligible at sizes above 1.2 fm
finite lattice spacing	considerable influence
unimproved action	reduced by stout smearing

- same influence of all of these effects:
  gluino-glue gets heavier than bosonic $a-\eta'$
- mass splitting is already hard to measure
  at $\beta = 1.75$ on a $24^3 \times 48$ lattice
- most important limitation: need large statistic,
  especially for the scalar particles (0^{++}, $a - f_0$)

$\Rightarrow$ large statistic at moderate lattice volume to estimate the effects of further improvements (extended stout, clover)