Supersymmetry on the lattice and the status of the SYM simulations

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1. No-Go for local lattice supersymmetry
2. Ginsparg-Wilson relation for SUSY
3. The simulations of SUSY Yang-Mills theory
4. Conclusions

Introduction

- supersymmetry is an important ingredient of many theories beyond the standard model
- the analysis of the quantum nature needs non-perturbative methods

basic properties of SUSY:

- nontrivial interplay between Poincare-symmetry and supersymmetry:
  $\left\{ Q_i \, , \, \bar{Q}_j \right\} = 2\delta_{ij}\gamma^{\mu} P_{\mu}$
- pairing of bosonic and fermionic states $\rightarrow m_f = m_b$

Wess-Zumino-models

matter sector of supersymmetric extensions of the standard model

Action:

$$S = \int d^2x \left[\frac{1}{2} \partial_\mu \phi \, \partial^\mu \bar{\phi} + \frac{1}{2} |W'(\phi)|^2 + \bar{\psi}(\not{\partial} + W''(\phi) P_+ + \bar{W}''(\bar{\phi}) P_-) \psi \right]$$

bos. potential from superpotential W

Yukawa from superpotential W

SUSY transformations:

$$\delta\phi = \bar{\psi}^1 \varepsilon_1 + \bar{\varepsilon}_1 \psi^1, \; \delta\bar{\psi}^1 = - \; \bar{W}'(\bar{\phi}) \; \bar{\varepsilon}_1 - \partial \phi \bar{\varepsilon}_2 \ldots$$

Variation of the action:

$$\delta S = -\bar{\varepsilon} \int dt \, [W'(\varphi)(\partial_t \psi) + \psi W''(\varphi) \partial_t \varphi] = -\bar{\varepsilon} \int dt \partial_t \, [\psi W'(\varphi)] = 0$$

Leibnizrule

boundary conditions

No-Go for local lattice supersymmetry

No Leibniz rule on the lattice:

For all lattice derivative operators ∇_{nm}:

$$\sum_m \nabla_{nm}(f_m g_m) - f_n \sum_m \nabla_{nm} g_m - g_n \sum_m \nabla_{nm} f_m \neq 0$$

possible way out: modification of lattice product

$$\int dx\phi^3 \to \sum_{i,j,k} C_{ijk}\phi_i\phi_j\phi_k$$

„No-Go theorem"[1]

To get SUSY at a finite lattice spacing a nonlocal derivative operator ∇_{nm} and a nonlocal product C_{ijk} is needed. (translational invariance assumed)

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Doubling problem as a second source of SUSY breaking

- Nielsen-Ninomiya theorem predicts doubling problem for all local ∇
- mismatch between fermionic and bosonic degrees of freedom or $m_f \neq m_b$

"Solutions":

1. bosonic doublers and Wilson mass for bosons in superpotential $W'(\phi)^2 = (m + m_w)^2\phi^2 + 2(mg + m_w g)\phi^3 + g^2\phi^4$
   $\Rightarrow$ nontrivial modification
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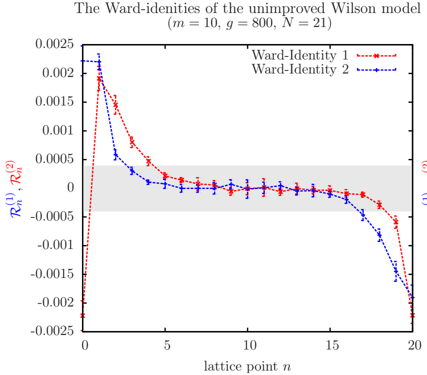
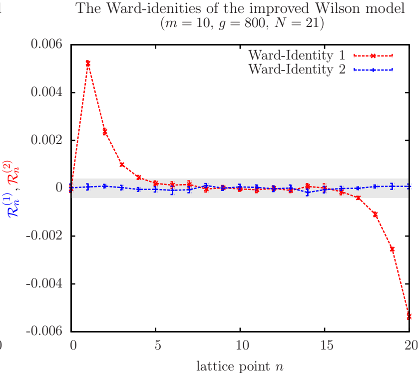
Partial realization of SUSY

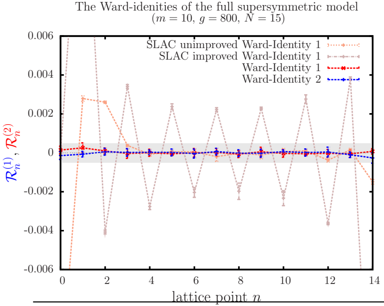




- techniques like Nicolai-improvement etc. allow realization of part of the SUSY
- not always improvement for complete SUSY
- Don't pay to much for only a part of the SUSY:
  violation of reflection positivity, unphysical contributions ...

Simulations with intact SUSY on the lattice

- only way: nonlocal product and derivative!
- perturbation theory[1] : local continuum limit in 1-3 dimensions
- simulation[2] : correct continuum limit for the masses



Result

It is possible to have a complete realization of SUSY on the lattice! New point of view: Instead of SUSY locality must be restored in continuum limit.

[1] [G.B., Kästner, Uhlmann & Wipf, Annals Phys.323:946-988,2008]
[Kadoh,Suzuki, Phys.Lett.B684:167-172,2010]

[2] [G.B., JHEP 1001:024,2010]

Generalization of the Ginsparg-Wilson relation

Situation seems similar to chiral symmetry, where the Ginsparg-Wilson relation led to a solution.
for arbitrary continuum symmetry: $S[(1+\varepsilon\tilde{M})\varphi] = S[\varphi]$:

Generalization of the Ginsparg-Wilson relation[1]

$$M^{ij}_{nm}\phi^j_m \frac{\delta S_L}{\delta\phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S_L}{\delta\phi^j_m}\frac{\delta S_L}{\delta\phi^i_n} - \frac{\delta^2 S_L}{\delta\phi^j_m \delta\phi^i_n}\right) + (\mathrm{STr} M - \mathrm{STr}\tilde{M})$$

provided

$$\int dx\, f(x-x_n)\,\tilde{M}^{ij}\varphi^j(x) = M^{ij}_{nm}\int dx\, f(x-x_m)\varphi^j(x)$$

[1] [GB, Bruckmann & Pawlowski, Phys.Rev.D79:115007,2009]

Solutions of the Ginsparg-Wilson relation for SUSY

Problems:

- M that follows from $\tilde{M}$ with derivative operator is nonlocal

$\hookrightarrow$ Poincare invariance not realized with GW approach

- solutions generically non-polynomial

$\hookrightarrow$ full effective action also non-polynomial

Possible solutions (currently under investigation):

- approximate relations: saddle-point approximations ...
- approximate solutions: truncations

The Veneziano-Curci approach: "brute force" SYM

SUSY Yang-Mills (λ adjoint Majorana fermion):

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

theory subject of many theoretical investigations of e. g.

- spontaneous symmetry breaking: $U_R(1) \stackrel{\text{anomaly}}{\longrightarrow} \mathbb{Z}_{2N_c} \stackrel{\langle\bar{\lambda}\lambda\rangle\neq 0}{\longrightarrow} \mathbb{Z}_2$
- domains, low energy effective actions, ...

Lattice action:

$$S_L = \beta\sum_P\left(1 - \frac{1}{N_c}\Re U_P\right) + \frac{1}{2}\sum_{xy}\bar{\lambda}_x\left(D_w(m_g)\right)_{xy}\lambda_y$$

- "brute force" discretization: Wilson fermions
- explicit breaking of symmetries: ~~chiral Sym.~~, ~~SUSY~~

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The Veneziano-Curci approach: Recovering symmetry

ward identity of chiral symmetry:

$$\langle \nabla^\mu J_\mu^{(A)}(x)\,\mathcal{O}\rangle = m_g \langle \bar{\lambda}(x)\gamma_5 \lambda(x)\,\mathcal{O}\rangle + \langle X^{(A)}(x)\mathcal{O}\rangle - \langle \delta(x)\mathcal{O}\rangle$$

renormalized, up to $O(a)$ [1]:

$$\langle \nabla^\mu Z^{(A)} J_\mu^{(A)}(x)\,\mathcal{O}\rangle = (m_g - \bar{m}_g)\langle \bar{\lambda}(x)\gamma_5 \lambda(x)\,\mathcal{O}\rangle - \langle \delta'(x)\mathcal{O}\rangle + \propto \langle F\tilde{F}\,\mathcal{O}\rangle$$

- tuning of m_g is enough for chiral limit
- Veneziano-Curci[2]: chiral limit = SUSY limit

[1] [Bochicchio et al., Nucl.Phys.B262:331,1985]

[2] [Veneziano, Curci, Nucl.Phys.B292:555,1987]

Setup for the simulations

- simulation algorithm: PHMC
- lattice sizes: 16^3x32, 24^3x48 (32^3x64)
- $r_0 = 0.5\text{fm} \rightarrow a \leq 0.088\text{fm}$; $L \approx 1.5 - 2.3\text{fm}$
- extrapolating to the chiral limit (connected $\bar{\lambda}\gamma_5\lambda$ vanishes ($m_{a-\pi}$)): SUSY Ward-identities vanish
- finite volume effects seem to be under control
- improvements: tree level Symanzik improved gauge action, stout smearing

SUSY Yang-Mills on the lattice I: Masses and multiplets

operators for correlators to obtain masses:

- adjoint mesons ($a-\eta'$: $\bar{\lambda}\gamma_5\lambda$, $a-f_0$: $\bar{\lambda}\lambda$)
- in SUSY limit mass disconnected contributions dominant
- glueball-like operators
- gluino-glue fermionic operator constructed from $\Sigma^{\mu\nu}\mathrm{Tr}[F_{\mu\nu}\lambda]$ ($F_{\mu\nu} \to$ clover plaquette)
- gluonic observables noisy: APE and Jacobi smearing

additional complication:

- reweighting the sign for Majorana-fermions: $det(D) \to \mathrm{Pf}(D)$ with $|\mathrm{Pf}(D)| = \sqrt{det(D)}$ (only relevant for a few configurations)

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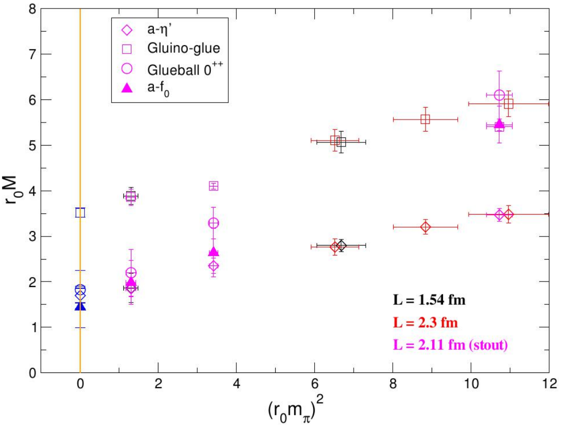
SUSY Yang-Mills results



No mass degeneracy in chiral limit, no change for larger volume!
⇒ smaller lattice spacing, further improvements

[Demmouche et al.,arXiv:1003.2073]

Conclusions

- No-Go for local lattice SUSY
- complete SUSY on the lattice: locality recovered for low dimensional Wess-Zumino models
- Ginsparg-Wilson relation for SUSY: local and polynomial action only with approximations
- fine-tuning “under control” for SYM but mass degeneracy not yet established (need further investigations)
- alternative method: FRG with error introduced by the truncation, but intact SUSY