Simulations of supersymmetric Yang-Mills theory

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Supersymmetry

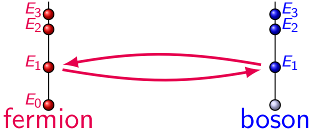
- supersymmetry is an important guiding principle for extensions of the standard model

fermionic operators Q

- fermions (spin halfinteger) $\longleftrightarrow$ bosons (spin integer)
- symmetry: $[Q, H] = 0$

$\Rightarrow$ pairing of states

- supersymmetry algebra: $\left\{ Q_i \, , \, \bar{Q}_j \right\} = 2\delta_{ij}\gamma^\mu P_\mu, \; \ldots$



Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- λ adjoint Majorana fermion
- $m_g \neq 0$ SUSY softly broken
- confinement
- low energy effective actions

multiplet[1]:
mesons : $a - f_0$: $\bar{\lambda}\lambda$; $a - \eta'$: $\bar{\lambda}\gamma_5\lambda$
fermionic gluino-glue $(\sigma_{\mu\nu}F_{\mu\nu}\lambda)$

multiplet[2]:
glueballs: 0^{++}, 0^{-+}
fermionic gluino-glue

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]
[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- "brute force" discretization: Wilson fermions

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger \right]$$

$$T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu}); \quad \kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$
- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

Ward identities on the lattice

Ward identities of supersymmetry and chiral symmetry:

$$\langle \nabla_\mu J_S^\mu(x)\,\mathcal{O}(y)\rangle = m_g \langle D_S(x)\,\mathcal{O}(y)\rangle + \langle X_S(x)\,\mathcal{O}(y)\rangle$$
$$\langle \nabla_\mu J_A^\mu(x)\,\mathcal{O}(y)\rangle = m_g \langle D_A(x)\,\mathcal{O}(y)\rangle + \langle X_A(x)\,\mathcal{O}(y)\rangle + \propto \langle F\tilde{F}\,\mathcal{O}\rangle$$

- classical (tree level): $X_S(x) = O(a)$, $X_A(x) = O(a)$

renormalization, operator mixing[1,2]:

$$\langle \nabla_\mu Z_A J_A^\mu(x)\,\mathcal{O}\rangle = (m_g - \bar{m}_g)\langle D_A(x)\,\mathcal{O}\rangle + \propto \langle F\tilde{F}\,\mathcal{O}\rangle + O(a)$$
$$\langle \nabla_\mu (Z_S J_S^\mu(x) + \tilde{Z}_S \tilde{J}_S{}^\mu(x))\,\mathcal{O}\rangle = (m_g - \bar{m}_g)\langle D_S(x)\,\mathcal{O}\rangle + O(a)$$

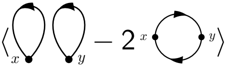
$\Rightarrow$ tuning of m_g: chiral limit = SUSY limit

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

Chiral limit

$$\langle \bar{\lambda}(x)\gamma_5\lambda(x)\,\bar{\lambda}(y)\gamma_5\lambda(y)\rangle = \langle$$



connected part: adjoint pion $(a-\pi)$

- disconnected part contains anomaly (OZI approximation)
- chiral limit: $m_{a-\pi}$ vanishes

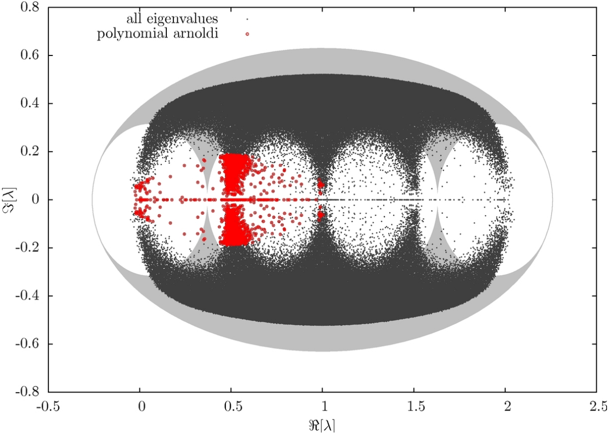
$\Rightarrow$ possible definition of gluino mass: $\propto (m_{a-\pi})^2$

At the end the consistency with the SUSY Ward identities is checked!

Specific challenges of SYM simulations

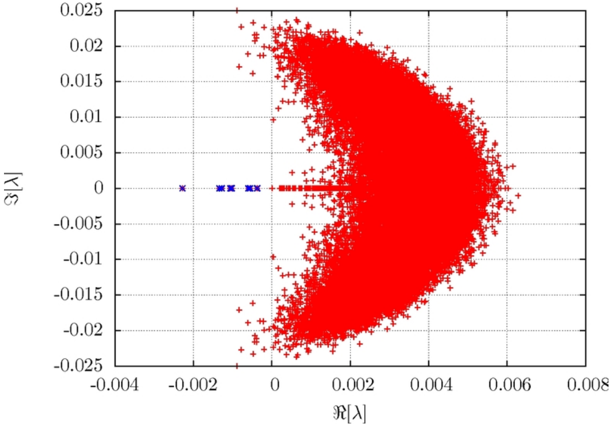
- Majorana fermion leads to $\mathrm{Pf}(D) = \mathrm{sign}(\mathrm{Pf}(D))\sqrt{\det(D)}$
- $\sqrt{\det(D)}$ using PHMC algorithm
- improvement of the polynomial approximation: reweighting factors from eigenvalues at small gluino masses
- reweighting with Pfaffian sign

Pfaffian sign $=$ sign($\prod$ real doubly degenerate eigenvalues))



efficient calculation of real eigenvalues: Arnoldi algorithm (ARPACK) + polynomial focussing and acceleration

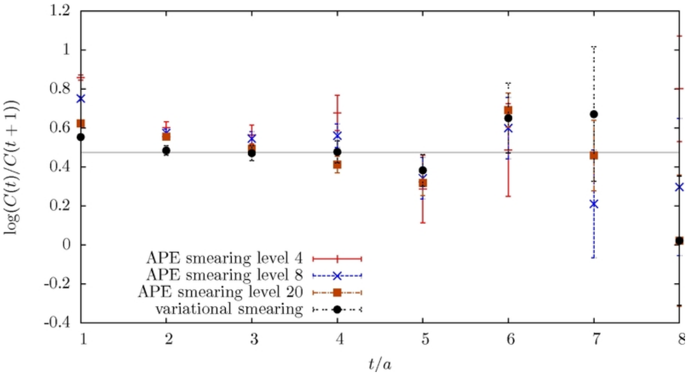
[G B., J. Wuilloud arXiv:1104.1363]



eigenvalues of D_w (2000 configurations at $\kappa = 0.1495$, $\beta = 1.75$) obtained with preconditioning and polynomial acceleration

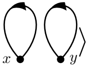
Observables I: Glueballs

- 0^{++}: correlator of simple combination of spatial plaquettes
- noisy: difficult to determine at larger Δt
- need to reduce overlap with excited states: smearing



Observables II: Meson operators

- disconnected contribution

$\langle$  $\rangle = \langle \mathrm{D}^{-1}(x,x)\,\mathrm{D}^{-1}(y,y)\rangle_{\text{eff}}$

- dominant at small gluino masses
- techniques: SET, IVST

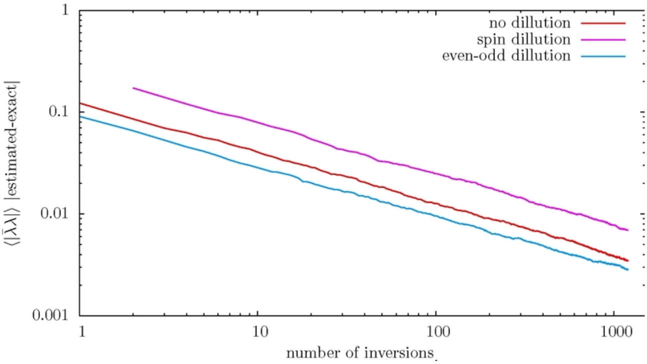
"stochastic estimator technique" (SET)

- random vectors $|\eta^i\rangle$ e. g. $\mathbb{Z}_4$
  $\frac{1}{N}\sum_{j=1}^{N}|\eta^j\rangle\langle\eta^j| := \overline{|\eta^j\rangle\langle\eta^j|} = \mathbb{1} + O\left(\frac{1}{\sqrt{N}}\right)$
- CG: $|s^j\rangle = M^{-1}|\eta^j\rangle$
- inverse $M^{-1} = \overline{|s\rangle\langle\eta|} + O\left(\frac{1}{\sqrt{N}}\right)$

$\Rightarrow$ improvements [Bali, Collins, Schäfer, Comput.Phys.Commun. 181 (2010)]

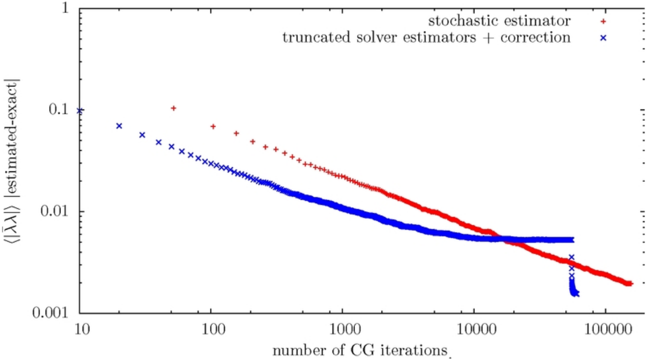
Improvements of SET: dilution

- sum of noisy estimators in subspaces to get complete inverse
- can reduce fluctuations



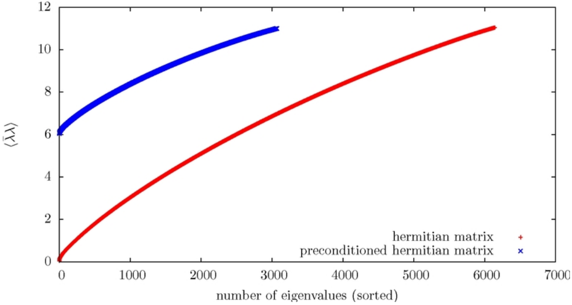
Improvements of SET: truncated solver method

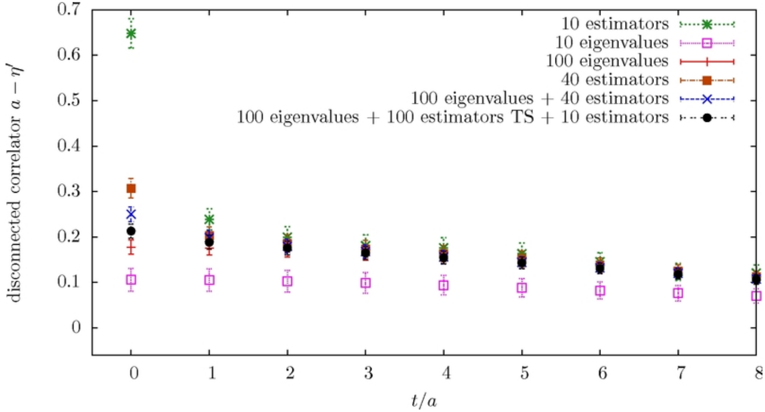
- large number of estimators with low precision in CG
- compensate accumulated error with precise correction steps



Improvements of SET: spectral decomposition

- exact contributions of N_e lowest eigenmodes of γ_5 D
- noise vectors projected orthogonal to eigenspace
- large improvment for $a-\eta'$ at small gluino masses
- preconditioned matrix, Chebyshev acceleration
- same eigenvalues can be used for reweighting factors!

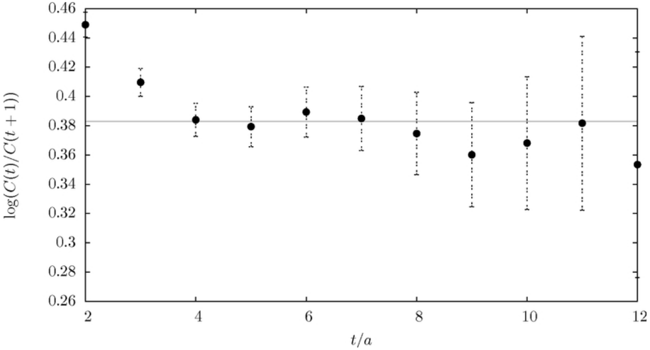




Time needed for the last three options is comparable!
(at small gluino masses)

Observables III: Gluino-Glue

- gluino-glue fermionic operator $\sigma^{\mu\nu}\mathrm{Tr}[F_{\mu\nu}\lambda]$
- $F_{\mu\nu}$ represented by clover plaquette
- APE smearing on gauge fields + Jacobi smearing on λ
- with combined smearing good signal compared to glueballs



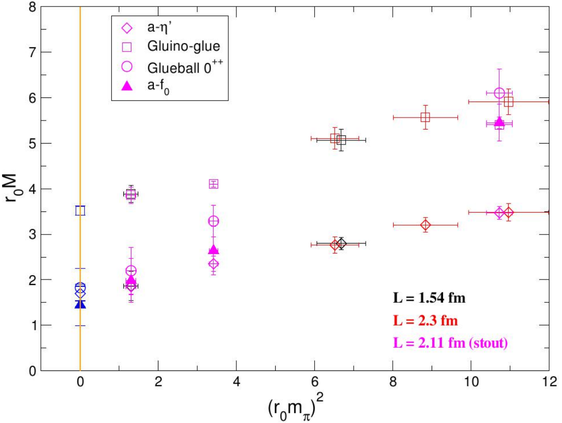
Details of the simulations

- simulation algorithm: PHMC
- tree level Symanzik improved gauge action
- stout smearing
- Sexton-Weingarten integrator
- determinant breakup

previous simulations:

- lattice sizes: 16^3x32, 24^3x48 (32^3x64)
- $r_0 \equiv 0.5\text{fm} \rightarrow a \leq 0.088\text{fm}$; $L \approx 1.5 - 2.3\text{fm}$

Previous SUSY Yang-Mills results



No mass degeneracy in chiral limit!
Tuning with SUSY Ward identities compatible with tuning of $m_{a-\pi}$. [Demmouche et al., Eur.Phys.J.C69 (2010)]

New simulations at smaller lattice spacing

Before speculating about new physics: Most likely explanation are lattice artifacts!

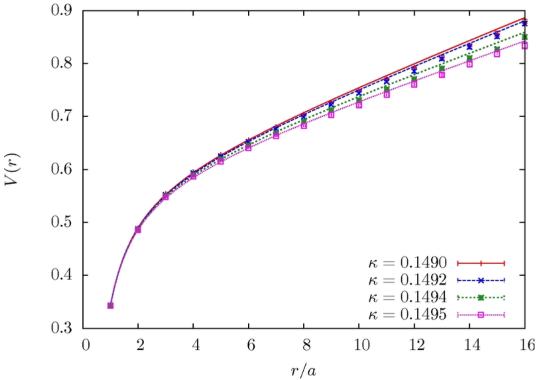
new simulations:

- volume fixed, smaller lattice spacing

$\Rightarrow$ increased β from 1.6 to 1.75

- simulations on $32^3 x 64$ lattice

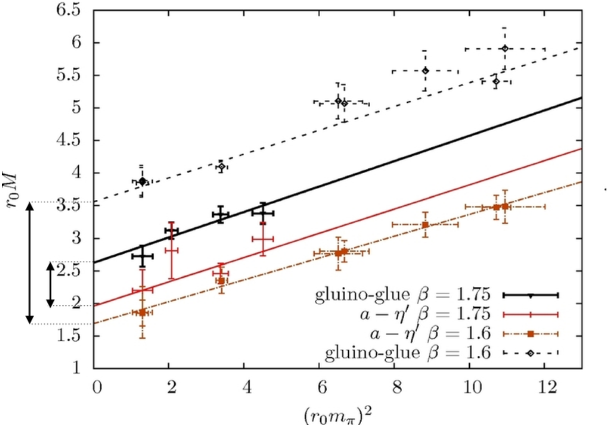
Confinement and physical scale of the new simulations



- good agreement with $V(r) = v_0 + c/r + \sigma r$ (confining)

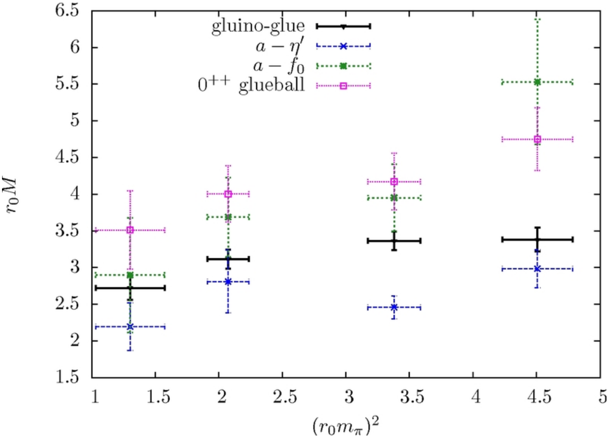
$\Rightarrow$ $a \approx 0.057\text{fm}$, $L \approx 1.8\text{fm}$

Comparison of the mass gap between $a - \eta'$ and gluino-glue



- mass gap considerably reduced
- gluino-glue has much lower mass

Complete spectrum obtained with the new simulations



- indicates mixing of $a-f_0$ and 0^{++} glueball
- in contrast to smaller lattice spacing: $a-f_0$, glueball heavier

Conclusions

- In supersymmetric Yang-Mills theory the unavoidable breaking of SUSY on the lattice can be controlled by a fine tuning of the gluino mass (κ).
- The simulations of this theory are challenging and advanced techniques must be used to get a reasonable signal of the observables.
- The spectrum was obtained in previous simulations, but the observed mass gap between bosonic and fermionic particles is not in accordance with supersymmetry.
- New simulations indicate that lattice artifacts are an explanation for this gap.
- Further simulations at a third, even smaller, lattice spacing can confirm these findings.