Effective lattice theory for finite temperature Yang Mills

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1. Strong coupling effective action approach
2. Polyakov line correlators in the effective theory
3. Thermodynamics of SU(3) effective PL theory
4. Conclusions

in collaboration with O. Philipsen, J. Langelage

Introduction

Approach

- effective description of finite temperature behaviour (confined phase)
- systematic derivation from the full Yang-Mills theory

Features

- better access to certain regions in parameter space
- tested also in heavy quark region
- results for finite chemical potential possible

Further reference

- talks by: J. Langelage, M. Neumann

This talk: compute/test further observables in Yang-Mills

Effective Polyakov loop action

$$e^{-S_{\text{eff}}[U_0]} = \int [dU_i] \prod_p e^{\frac{\beta}{6}\text{Tr}\left(U_p + U_p^\dagger\right)}$$

- integrating out spatial links U_i
- dimensional reduction from $3+1$D to $3D$
  $U_\mu(x,t) \to U_0(x) \to$ Polyakov lines $L(x)$
- no complete calculation possible
  $\Rightarrow$ organization of interactions in S_{eff} e.g. ordered by distance
- several approaches: inverse MC, demon methods [Heinzl, Kästner, Wozar, Wipf, Wellegehausen], relative weights [Langfeld, Greensite], ...

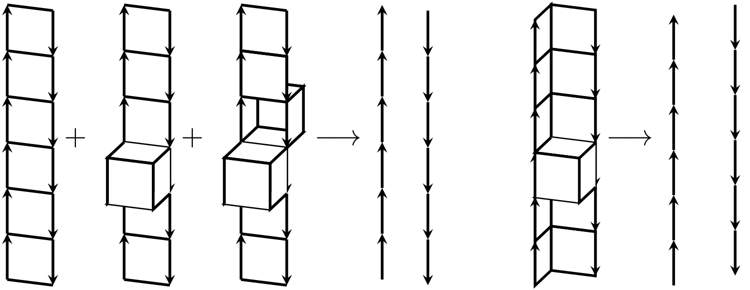
$$Z = \int [dL] e^{-S_{\text{eff}}[L]}$$

Effective action from strong coupling

- character expansion:

$$e^{\frac{\beta}{6}\mathrm{Tr}\left(U_p+U_p^\dagger\right)} = \sum_{r\in irreps.} (1 + d_r a_r(\beta)\chi_r(U_p))$$

- expansion parameter $u = a_f$ (resummation)
- cluster expansion



[Polonyi, Szlachanyi]

Effective action from strong coupling and simulations

$$S_{\text{eff}} = \lambda_1 S_{\text{nearest neighbors}} + \lambda_2 S_{\text{next to nearest neighbors}} + \dots$$

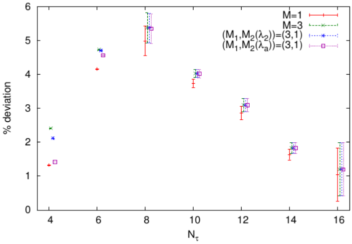
- ordering principle for the interactions
  higher representations and long distances are suppressed
  $(u^{N_t};\ u^{2N_t};\ u^{2N_t+2})$
- effective couplings exponentiate:
  $\lambda_1 = u^{N_t} \exp(N_t P(u))$ (resummation)
- collect similar terms to log (resummation)

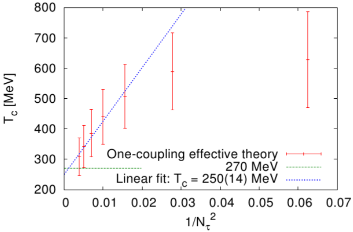
$$S_{\text{nearest neighbors}} = \sum_{<ij>} (\lambda_1 \Re L_i L_j^* - (\lambda_1 \Re L_i L_j^*)^2 + \dots)$$
$$= \sum_{<ij>} \log(1 + \lambda_1 \Re L_i L_j^*)$$

Simulations of the effective theory

Non-perturbative effects from MC simulation of effective theory.

- as in pure SU(3) YM: 1st order phase transition, spont. broken centre symmetry
- higher representations, long distances suppressed in continuum limit
- $(\lambda_1)_c$ mapped back to $(\beta_c)_{\mathbf{eff}} \to T_c$
- few percent difference $(\beta_c)_{\mathbf{eff}}$ to $(\beta_c)_{\mathbf{YM}}$



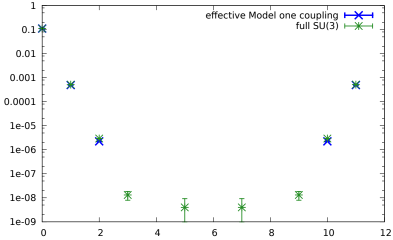


Precise test of strong coupling approach

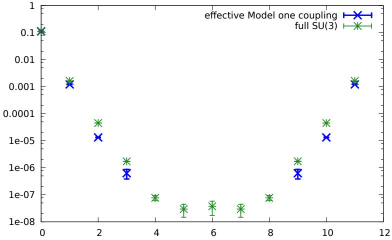
- Polyakov line correlator
  $\langle L(\vec{0})L^\dagger(\vec{R})\rangle$ good test for effective actions
- related to free energy in presence of heavy quarks
  $\langle L(\vec{0})L^\dagger(\vec{R})\rangle = \exp(-F(|\vec{R}|, T)/T)$
- continuum: depends only on $|\vec{R}|$;
  lattice: dependence on the direction
  (breaking of rotational symmetry)
- sign for the restoration of rotational symmetry in the continuum limit
- precise check

Polyakov loop correlator

$\beta = 5.0,\ N_t = 6$



$\beta = 5.4,\ N_t = 6$



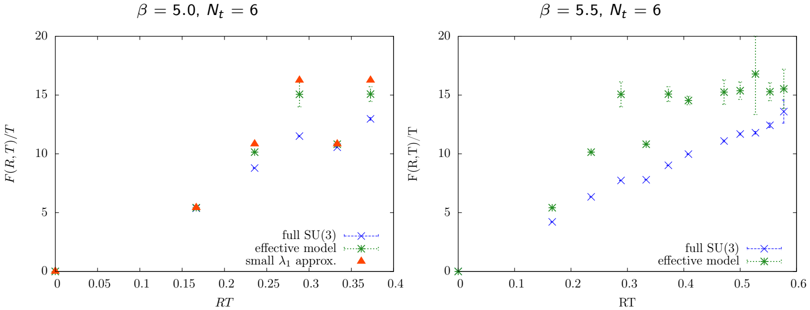
- YM strong coupling region: multilevel and mulithit algorithm
- deviations close to $(\beta_c)_{\mathbf{YM}}$; but still reasonable agreement
- larger deviations in off-axis correlator
- next to nearest neighbor interactions: small improvement

Strong coupling off-axis correlator

Small λ_1 behaviour: $F(R/a, T)/T = d(R/a) N_t C(\beta)$

- $d(R/a)$ smallest number of lattice spacings connecting points with distance R/a on the lattice
- breaking of rotational symmetry as in strong coupling YM
- no UV $1/r$ part at small λ_1

$\Rightarrow$ approximates strong coupling correlators

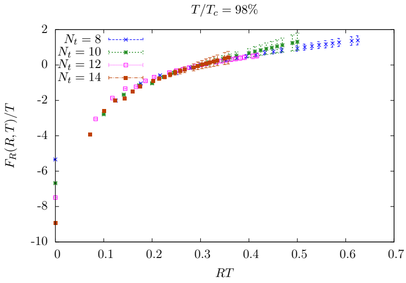


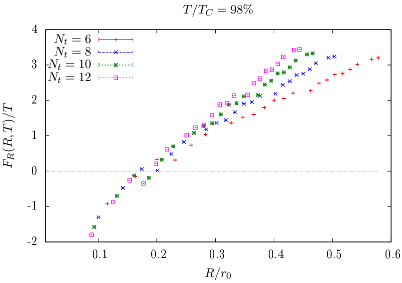
Polyakov loop correlator and continuum limit

continuum behaviour at large λ_1:

effective model close to $(\beta_c)_{\text{eff}} \leftrightarrow$ YM close to $(\beta_c)_{\text{YM}}$

- both: restoration of rotational symmetry
- YM: scaling behaviour of renormalized correlator
- effective model: still need identify scaling region $(\sqrt{\sigma}/T)$ (using scale setting of YM)



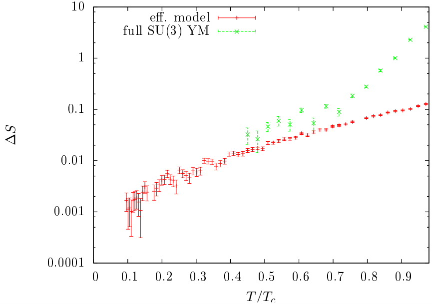


Effective theory and thermodynamics

primary observable:

$$\frac{d}{d\beta}\frac{p}{T^4} = \Delta S$$

- better agreement with YM than strong coupling expansion
- useful to get small T/T_c results



$N_\tau = 4$

Conclusions

- systematic derivation of effective PL theory:
  strong coupling series
- non-perturbative simulations of effective theory: reasonable agreement with full theory in confined phase in contrast to strong coupling results
- towards continuum limit higher orders in the expansion are important
- Can we identify intermediate scaling region?

- [x] T_c from $(\lambda_1)_c$
- [ ] Polyakov loop correlators: must be outside perturbative region of effective theory $\Rightarrow$ close to $(\lambda_1)_c$, below certain N_t
  open issue: scale setting / renormalization in effective theory
- [ ] improved strong coupling results also for thermodynamics