Testing effective Polyakov loop actions derived form a strong coupling expansion

Georg Bergner
ITP GU Frankfurt

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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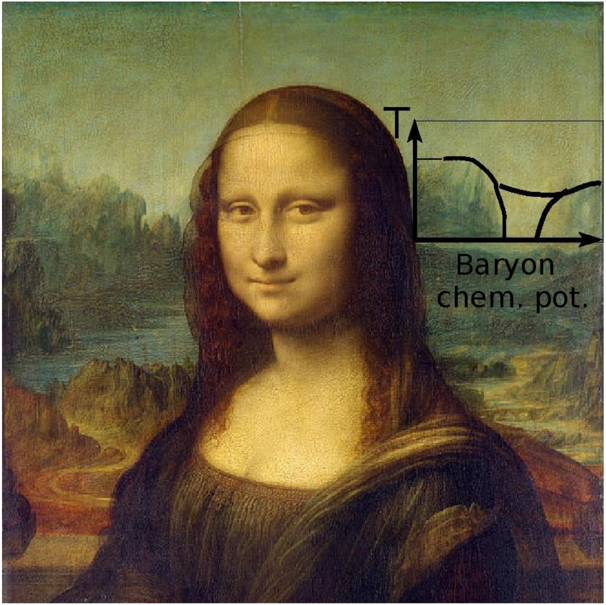
Bundesministerium für Bildung und Forschung in collaboration with O. Phillipsen, J. Langelage, S. Lottini, W. Unger, M. Neuman, M. Fromm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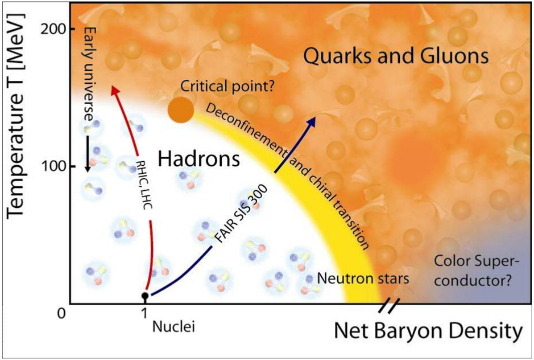
chem. pot.

The final goal

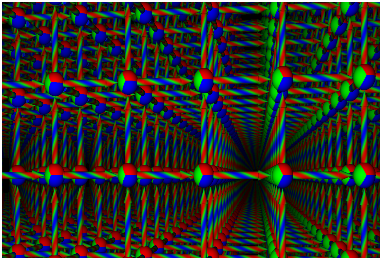


- critical temperature: confinement $\rightarrow$ deconfinement
- critical temperature: chiral symmetry restoration
- properties of the phases: $\epsilon(T)$, $p(T)$, screening length, ...

QCD on the lattice

$$Z = \int \mathcal{D}\phi \; e^{-S[\phi]} = \int \prod_i d\phi_i \; e^{-S[\phi]}$$

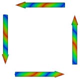
- discretized continuum action
- non-perturbative computations
- offers different expansion schemes
  ("opposite of" weak coupling perturbation theory)



gauge fields
$A_\mu \to e^{igaA_\mu} = U_\mu$:
matter fields ψ, ϕ:
$T = \frac{1}{L_t} = \frac{1}{aN_t}$

QCD on the lattice: the action

$$\mathcal{L} = \beta \sum_p \left(1 - \frac{1}{3}\Re(\mathrm{Tr}\, U_p)\right) + \sum_f \bar{\psi}_f (\mathrm{D[U]} + m_f)\psi_f$$

- plaquette $U_p =$ 
- integral of group elements: Haar measure dU
- integral of Grassmann fields: integrated out

$$Z = \int \prod_i dU_i \prod_f \det(D[U] + m_f) \exp(-S_g[U])$$

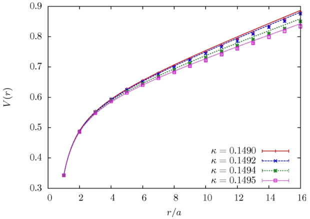
QCD on the lattice: fermions

$$\sum_x \bar\psi(x)(\mathrm{D}-m)_{x,y}\psi(y) = \sum_x \Big[(m+4r)\bar\psi(x)\psi(x)$$
$$+\frac{1}{2}\sum_\mu \bar\psi(x)\big((\gamma_\mu - r)U_\mu(x)\psi(x+\hat\mu) + (\gamma_\mu + r)U^\dagger_\mu(x-\hat\mu)\psi(x-\hat\mu)\big)\Big]$$

- lattice Dirac operator D: derivatives replaced by gauge invariant difference operators
- hopping parameter $1+\kappa\mathrm{H}$ with $\kappa = \frac{1}{2m+8r}$
- spacial (U_i) and temporal (U_0) hops
- Wilson-Dirac operator: additional momentum dependent mass term $\Rightarrow$ chiral symmetry breaking
- fine tuning $\kappa \to \kappa_c$: chiral continuum limit

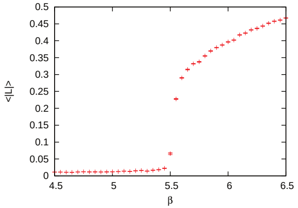
Confinement/Deconfinement

static quark-antiquark potential:



- confinement:
  linear rise at large distance r:
  $V(r) = -\frac{c}{r} + \sigma r$
- deconfinement:
  Yukawa type screening potential

Polyakov loop: $L(x) = \mathrm{Tr}\, W = \mathrm{Tr}[\prod_{t=0}^{N_t} U_0(t,x)]$



- L puts infinitely heavy quark in the theory
- $\langle L \rangle = \exp(-(F_Q - F_0)/T)$
- confinement $F_Q \to \infty$: $\langle L \rangle = 0$
- deconfinement: $\langle L \rangle \neq 0$,
  L around center elements

Lattice QCD and finite density

$$Z(T,\mu) = \mathrm{Tr}(e^{-(H-\mu Q)/T})$$

- continuum physics: extra term $\mu\bar{\psi}\gamma_0\psi$
- on the lattice modification of D

$$\bar{\psi}(x)\big((\gamma_0 - r)e^{a\mu}U_0(x)\psi(x+\hat{0}) + (\gamma_0 + r)e^{-a\mu}U_0^\dagger(x-\hat{0})\psi(x-\hat{0})\big)$$

- $\gamma_5\mathrm{D}(\mu)^\dagger\gamma_5 = \mathrm{D}(-\mu^*) \Rightarrow \det(\mathrm{D}(\mu)) = \det(\mathrm{D}(-\mu^*))^*$
- complex determinant
- all methods fail at large μ

$\Rightarrow$ any information about the model at finite μ is helpful, even in unphysical limit.

$\Rightarrow$ need playground to test methods and find possible effects.

Strong coupling expansion in lattice gauge theory

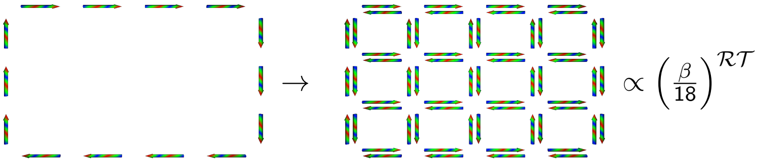
$$Z = \int [dU_\mu] \prod_p e^{\frac{\beta}{6}\mathrm{Tr}\left(U_p + U_p^\dagger\right)}$$

- expansion in $\beta = 6/g^2$ (opposite to weak coupling)
- similar to high temperature expansion in statistical physics
- simple integration rules for products of plaquette contributions

$$\int dU\; U = \int dU\; U^\dagger = 0; \quad \int dU\; UU^\dagger = \frac{1}{3}\mathbb{1}$$

Static quark-antiquark potential in strong coupling limit

simplest example: $\langle$ Wilson loop $\rangle$



- first contribution: Loop filled with plaquettes
- confinement:
  $V(\mathcal{R}) = -\lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \log\langle W\rangle = -\sigma\mathcal{R}$
- extension: $O = \sum_n O_n \beta^n$
- in certain region convergent series

Effective action for the Polyakov loop

$$e^{-S_{\text{eff}}[U_0]} = \int [dU_i] \prod_p e^{\frac{\beta}{6}\mathrm{Tr}\left(U_p + U_p^\dagger\right)}$$

- integrating out spatial links
- final result depends only on Polyakov lines L
- dimensional reduction from $3+1$D to $3D$
  $U_\mu(x,t) \to U_0(x) \to L(x)$
- no complete calculation possible
  $\Rightarrow$ expansion of S_{eff} e.g. in terms of interaction distance
- several ways to calculate it: inverse MC, demon methods [Heinzl, Kästner, Wozar, Wipf, Wellegehausen], relative weights [Langfeld, Greensite], ...

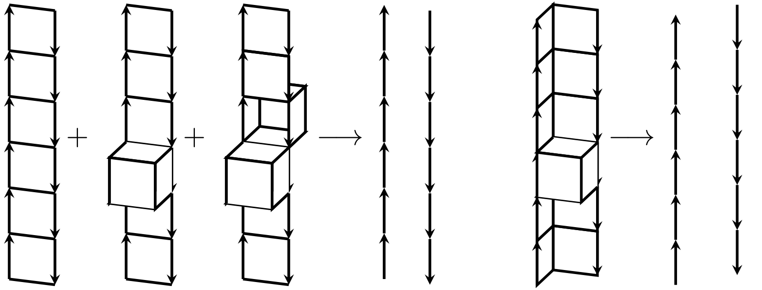
$$Z = \int [dL] e^{-S_{\text{eff}}[L]}$$

Effective action from strong coupling

- character expansion:

$$e^{\frac{\beta}{6}\mathrm{Tr}\left(U_p+U_p^\dagger\right)} = \sum_{r\in irreps.} \left(1 + d_r a_r(\beta)\chi_r(U_p)\right)$$

- expansion parameter $u = a_f$ (resummation)
- cluster expansion



[Polonyi, Szlachanyi]

Effective action from strong coupling and simulations

$$S_{\text{eff}} = \lambda_1 S_{\text{nearest neighbors}} + \lambda_2 S_{\text{next to nearest neighbors}} + \dots$$

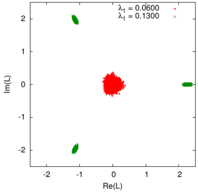
- ordering principle for the interactions
  higher representations and long distances are suppressed
  $(u^{N_t};\ u^{2N_t};\ u^{2N_t+2})$
- effective couplings exponentiate:
  $\lambda_1 = u^{N_t} \exp(N_t P(u))$ (resummation)
- collect similar terms to log (resummation)

$$S_{\text{nearest neighbors}} = \sum_{<ij>} (\lambda_1 \Re L_i L_j^* - (\lambda_1 \Re L_i L_j^*)^2 + \dots)$$

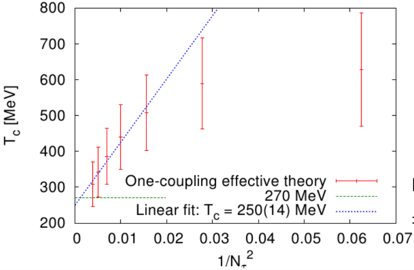
$$= \sum_{<ij>} \log(1 + \lambda_1 \Re L_i L_j^*)$$

Results in pure Yang-Mills obtained with the effective action

- observables from nonperturbative MC simulation of S_{eff} (resummation)



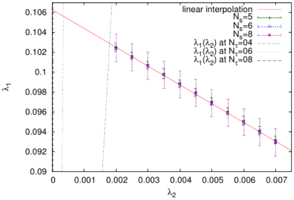
- confinement/deconfinement phase transition

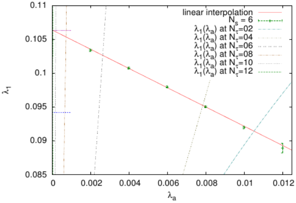


mapping back to β_c
$\Rightarrow T_c$ (continuum limit)

Quality of the results

Next to nearest neighbors, adjoint rep. ...

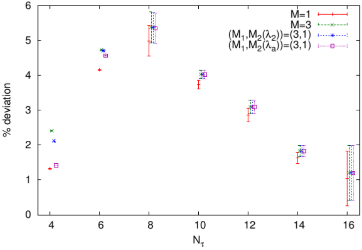




... are not important for the phase transition in continuum limit.

β_c relative error effective theory

$\Rightarrow$ good agreement



Quarks in the effective action

$$S_q = -\log\left[\prod_f \det(D-m)\right] = -N_f \mathrm{Tr}\log(1-\kappa H) = N_f \sum_l \frac{\kappa^l}{l!}\mathrm{Tr} H^l$$

- $\Rightarrow$ truncate hopping parameter expansion
- simplest contributions (no spacial hops): single Polyakov lines (L,L^*)
- integrating out spacial links in strong coupling expansion
- resummation $\exp(-S_q) \rightarrow \prod_x \det()$

$$\prod_x \det((1+hW(x))(1+\bar{h}W^*(x)))^{2N_f}$$

$$= \prod_x \left[(1+hL(x)+h^2L^*(x)+h^3)(1+\bar{h}L^*(x)+\bar{h}^2L(x)+\bar{h}^3)\right]^{2N_f}$$

[de Pietri, Feo, Seiler, Stamatescu]

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[de Pietri, Feo, Seiler, Stamatescu]

Quarks in the effective action

general form of the action

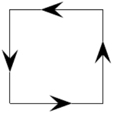
$$\sum_i \lambda_i S_{\text{symm.}i} + \sum_i h_i S_{\text{asymm.}i} + \sum_i \bar{h}_i S^\dagger_{\text{asymm.}i}$$

- center symmetric part of the action similar to pure Yang-Mills
- fermions introduce asymmetric contributions
- finite μ introduces factor $e^{\pm a\mu}$ for temporal up/down hops $\Rightarrow h \neq \bar{h}$
- $h(\mu) = \bar{h}(-\mu) \Rightarrow$ sign problem
- sign problem is mild (reweighting works in large region)
- alternative algorithm: Worm algorithm
- alternative algorithm: complex Langevin

Quarks to the effective action

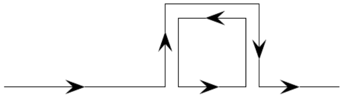
complicated pattern at higher order

- quark lines building up plaquette like objects



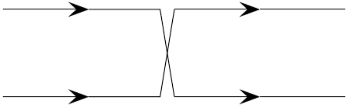
$\Rightarrow \lambda(\beta, \kappa)$ (shift of β)

- plaquette contributions to quark lines



$\Rightarrow h(\kappa, \beta)$

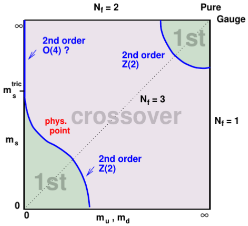
- interaction between quark lines

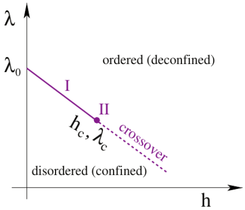


$\Rightarrow L_i L_j$ interaction

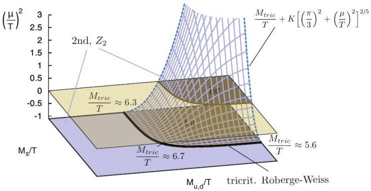
Results with quark matter and finite density

- reproduce phase transition in heavy quark limit



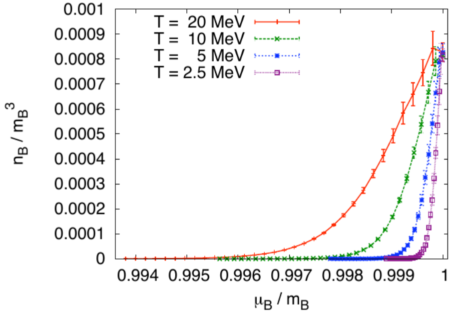


- phase transition at finite densities



Results with quark matter and finite density

Nuclear transition:



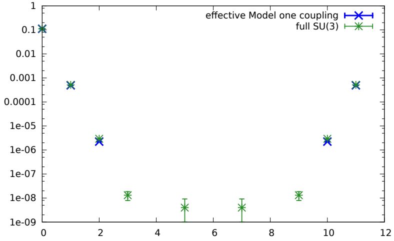
$\Rightarrow$ limited to $m_B \approx 30$ GeV

Further test of this program

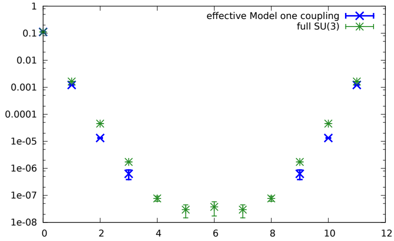
- Polyakov line correlator
  $\langle L(\vec{0}) L^\dagger(\vec{R}) \rangle$ good test for effective actions
- related to static quark potential
  $\langle L(\vec{0}) L^\dagger(\vec{R}) \rangle = \exp(-V(|\vec{R}|, T)/T)$
- continuum: depends only on $|\vec{R}|$;
  lattice: dependence on the direction
  (breaking of rotational symmetry)
- sign for the restoration of rotational symmetry in the continuum limit
- precise check [Greensite, Langfeld];
  some nonperturbative methods might fail

Test for the diagonal correlator

$\beta = 5.0$

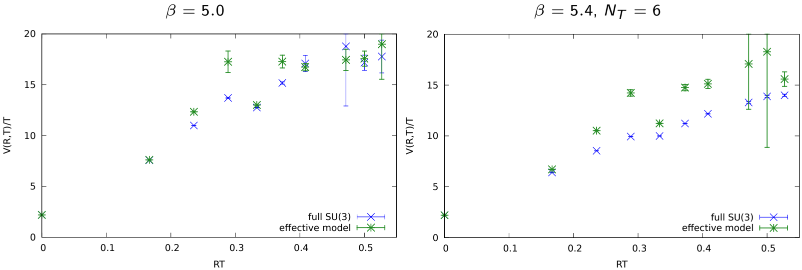


$\beta = 5.4,\ N_T = 6$



- results obtained with Multilevel and Mulithit algorithm
- to include strong coupling region:
  need error below 10^{-9}
- deviations close to critical β; but still reasonable agreement

Test for the off diagonal correlator



- off diagonal terms remain closer to strong coupling behavior
- less restoration of rotational symmetry
- small improvement with next to nearest neighbor interactions
- need more information about continuum limit

Conclusions and outlook

- effective theories can be derived from a strong coupling series
- includes explicit ordering principle,
  higher orders suppressed with smaller β, larger N_t
- reproduce phase transitions of pure Yang-Mills theory
- quarks included in hopping parameter expansion
- most important current limitation:
  truncation of the κ series
  $\Rightarrow$ higher orders included
- limits N_t for the confinement/deconfinement transition
- precise check: L correlator
- need to understand the suppression of the interactions at larger distances in the continuum limit