Fits to $K_{\ell 4}$ data and $N_f = 2$ χPT

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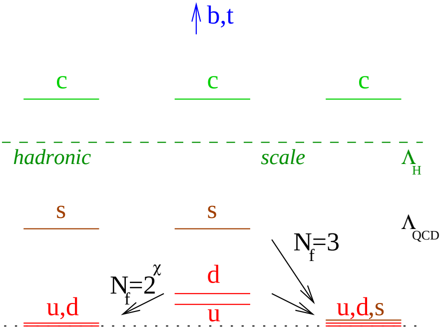
6 March 2007

Three chiral limits of interest



$m_u, m_d \to 0$

$N_f = 3$: $m_s \to 0$
$N_f = 2$: m_s physical
$N_f = 2^{\text{lat}}$: no dynamical s

Two versions of χPT	$N_f = 2$:	π only d.o.f	(few param. & processes)
	$N_f = 3$:	π, K, η d.o.f	(more param. & processes)

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \to 0} -\langle 0|\bar{u}u|0\rangle$$

$$\begin{cases} \Sigma(3) & = & \Sigma(2; 0) \\ \Sigma(2) & = & \Sigma(2; m_s^{\mathrm{phys}}) \\ \Sigma(2^{\mathrm{lat}}) & = & \Sigma(2; \infty) \end{cases}$$

From 2 to 3 massless flavours

$$\Sigma(2;m_s) = \lim_{m_u,m_d\to 0} -\langle 0|\bar{u}u|0\rangle \qquad \left\{ \begin{array}{rcl} \Sigma(3) & = & \Sigma(2;0) \\ \Sigma(2) & = & \Sigma(2;m_s^{\rm phys}) \\ \Sigma(2^{\rm lat}) & = & \Sigma(2;\infty) \end{array} \right.$$

$$\Sigma(2;m_s) = \Sigma(2;0) + m_s \frac{\partial\Sigma(2;m_s)}{\partial m_s} + O(m_s^2)$$

From 2 to 3 massless flavours

$$\Sigma(2;m_s) = \lim_{m_u,m_d\to 0} -\langle 0|\bar{u}u|0\rangle \qquad \left\{ \begin{array}{rcl} \Sigma(3) & = & \Sigma(2;0) \\ \Sigma(2) & = & \Sigma(2;m_s^{\rm phys}) \\ \Sigma(2^{\rm lat}) & = & \Sigma(2;\infty) \end{array} \right.$$

$$\Sigma(2) = \Sigma(3) + m_s^{\rm phys} \lim_{m_u,m_d\to 0} i\int d^4x\ \langle 0|\bar{u}u(x)\,\bar{s}s(0)|0\rangle + O(m_s^2)$$

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \to 0} -\langle 0 | \bar{u} u | 0 \rangle \qquad \left\{ \begin{array}{rcl} \Sigma(3) & = & \Sigma(2; 0) \\ \Sigma(2) & = & \Sigma(2; m_s^{\rm phys}) \\ \Sigma(2^{\rm lat}) & = & \Sigma(2; \infty) \end{array} \right.$$

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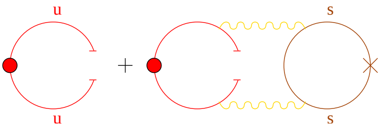
$\Sigma(2)$ contains

- A "genuine" condensate $\Sigma(3)$
- An "induced" condensate $m_s \times$ (scalar N_c-suppressed)
  [effect from sea $s\bar{s}$-pairs]



$\Sigma(2)$ and $\Sigma(3)$

- Theory: $\Sigma(3) \leq \Sigma(2)$ but relative size ? $\quad X(N_f) = \frac{2m\Sigma(N_f)}{F_\pi^2 M_\pi^2}$
- Experimentally : $\Sigma(2) \leftrightarrow \pi$ physics $\quad \Sigma(3) \leftrightarrow K, \eta$ physics

$$\text{Three possibilities} \begin{cases} \Sigma(3) \simeq \Sigma(2) \text{ and large (common lore)} \\ \Sigma(3) \simeq \Sigma(2) \text{ and small} \\ \Sigma(3) \text{ small and } \Sigma(2) \text{ large} \end{cases}$$

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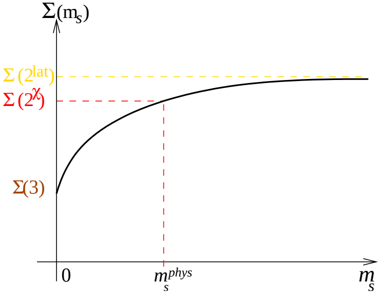


$\Sigma(3) \simeq \Sigma(2)$ and $\langle(\bar{u}u)(\bar{s}s)\rangle$ small
Zweig rule OK for scalars
No impact of strange sea quarks

or

$\Sigma(3) < \Sigma(2)$ and $\langle(\bar{u}u)(\bar{s}s)\rangle$ large
Large Zweig-rule violation
Strange sea quarks important

$\pi\pi$ scattering

$$\langle \pi^a \pi^b | T | \pi^c \pi^d \rangle = \delta^{ab}\delta^{cd} A(s|t,u) + \delta^{ac}\delta^{bd} A(t|s,u) + \delta^{ad}\delta^{bc} A(u|s,t)$$

Unitarity
Analyticity $\Longrightarrow A(s|t,u) = A_{\mathrm{cut}}(s|t,u) + A_{\mathrm{pol}}(s|t,u) + O(p^8)$
Crossing sym.

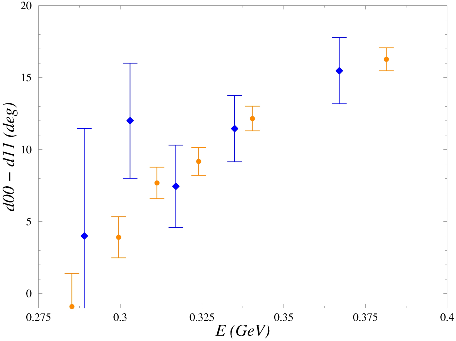
M.Knecht,B.Moussallam,J.Stern,N.Fuchs

- A_{cut}: cuts due to unitarity and crossing symmetry,
- A_{pol}: $t \leftrightarrow u$ symmetric polynomial with 6 subthreshold parameters.

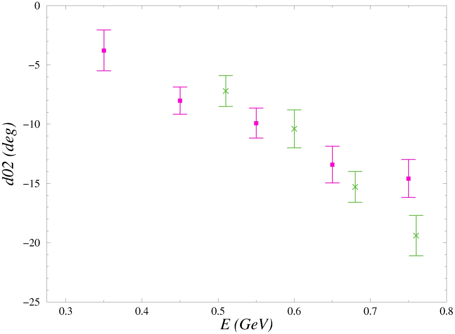
$$\begin{aligned} A_{\mathrm{pol}}(s|t,u) &= \frac{\alpha_{\pi\pi}}{F_\pi^2}\frac{M_\pi^2}{3} + \frac{\beta_{\pi\pi}}{F_\pi^2}\left(s - \frac{4M_\pi^2}{3}\right) \\ &+ \frac{\lambda_1}{F_\pi^4}(s-2M_\pi^2)^2 + \frac{\lambda_2}{F_\pi^4}\left[(t-2M_\pi^2)^2 + (u-2M_\pi^2)^2\right] \\ &+ \frac{\lambda_3}{F_\pi^6}(s-2M_\pi^2)^3 + \frac{\lambda_4}{F_\pi^6}\left[(t-2M_\pi^2)^3 + (u-2M_\pi^2)^3\right] \end{aligned}$$

Experimental information

$K \to \pi\pi e\nu$ decay
$\delta_0^0 - \delta_1^1$
Rosselet, E865



$\pi^+\pi^+$ prod.
δ_0^2
Hoogland, Losty



Roy equations

T^I s-channel isospin amplitudes, with partial-wave decomposition

$$T^I(s,t) = 32\pi \sum_\ell (2\ell+1) P_\ell \left(1 + \frac{2t}{s-4M_\pi^2}\right) t_\ell^I(s)$$

$$t_\ell^I(s) = \frac{1}{2i}\sqrt{\frac{s}{s-4M_\pi^2}} \left[\eta_\ell^I(s) \exp\left[2i\delta_l^I(s)\right] - 1\right]$$

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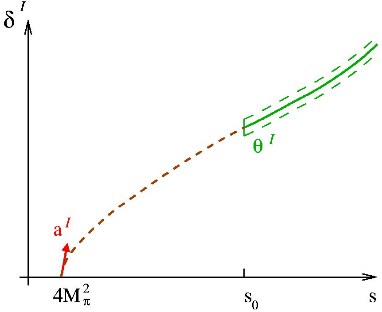
At low energies, only $(I,\ell)=(0,0),(1,1),(2,0)$, following Roy equations

$$\mathrm{Re}\ t^I(s) = k^I(s) + \sum_{I'=0,2}\int_{4M_\pi^2}^{s_0} ds' K^{I,I'}(s,s')\mathrm{Im}\ t^{I'}(s) + \sum_{I'=0,2}\int_{s_0}^{s_2} ds' K^{I,I'}(s,s')\mathrm{Im}\ t^{I'}(s) + d^I(s)$$

- k^I subtraction term, function of s and (a_0^0, a_0^2)
- $K^{I,I'}(s,s')$ known integral kernels, $\sqrt{s_0}=0.8$ GeV and $\sqrt{s_2}=2$ GeV
- d^I known driving terms (higher partial waves, $s \geq s_2$)

Numerical solutions of Roy equations



Roy equations yield a boundary value problem:

$$\begin{array}{c}(a_0^0, a_0^2)\\ \mathrm{Im}\ t^I \text{ for } s_0 \le s \le s_2\\ \Downarrow\\ \delta_0^0, \delta_1^1, \delta_2^2 \text{ for } 4M_\pi^2 \le s \le s_0\end{array}$$

- Analysis and numerical solutions for fixed values of $\theta^I = \delta^I(s_0)$
  B.Ananthanarayan,G.Colangelo,J.Gasser,H.Leutwyler
- Similar analysis allowing θ^I to vary within their experimental range.
  SDG,N.Fuchs,L.Girlanda,J.Stern

Extraction of (a_0^0, a_0^2)

$\delta_0^0 - \delta_1^2$ not enough to fix 2 parameters, we need more !

Two fits with E865+Rosselet, Hoogland+Losty

Global	ACGL sols. of Roy eqs.	fixed $\theta_{0,1}$
Extended	our sols. of Roy eqs.	free $\theta_{0,1}$

SDG,N.Fuchs,L.Girlanda,J.Stern

One scalar fit: E865 + standard χPT constraint from

- dispersive estimate of pion scalar radius $\langle r\rangle_\pi^s$
- large-N_c estimate of $O(p^6)$ LECs in the scalar sector

G.Colangelo,J.Gasser,H.Leutwyler

$N_f = 2$ LECs and order parameters: E865 data

From (a_0^0, a_0^2) and numerical solutions of Roy equations,

- Reconstruct $A(s|t,u)$ in subthreshold region to extract α, β

$N_f = 2$ LECs and order parameters: E865 data

From (a_0^0, a_0^2) and numerical solutions of Roy equations,

- Reconstruct $A(s|t,u)$ in subthreshold region to extract α, β
- Relate (α, β) to $N_f = 2$ LECs $(\bar{\ell}_3, \bar{\ell}_4)$

$$
\begin{aligned}
\alpha &= 1 - (1 + 3\bar{\ell}_3 - 4\bar{\ell}_4)\xi Y(2)^2/2 + \delta_\alpha \,, \\
\beta &= 1 + 2(\bar{\ell}_4 - 1)\xi Y(2) + \varepsilon_\beta \,.
\end{aligned}
$$

where $Y(2) = 2mB/M_\pi^2 =$ non-linear $f(\bar{\ell}_3, \bar{\ell}_4)$
taking into account scenarios of small and large $\Sigma(2)$

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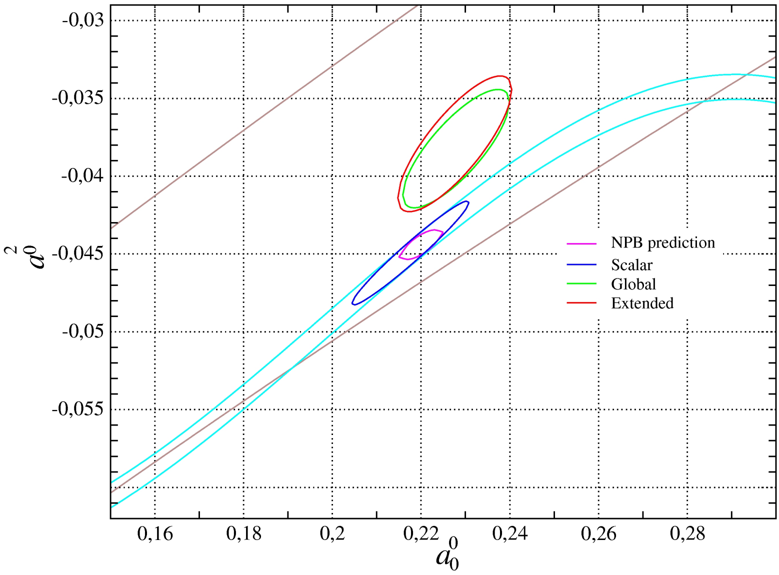
where $Y(2) = 2mB/M_\pi^2 =$ non-linear $f(\bar{\ell}_3, \bar{\ell}_4)$
taking into account scenarios of small and large $\Sigma(2)$

- Relate $(\bar{\ell}_3, \bar{\ell}_4)$ to $N_f = 2$ order param. (quark condensate, decay cst.)

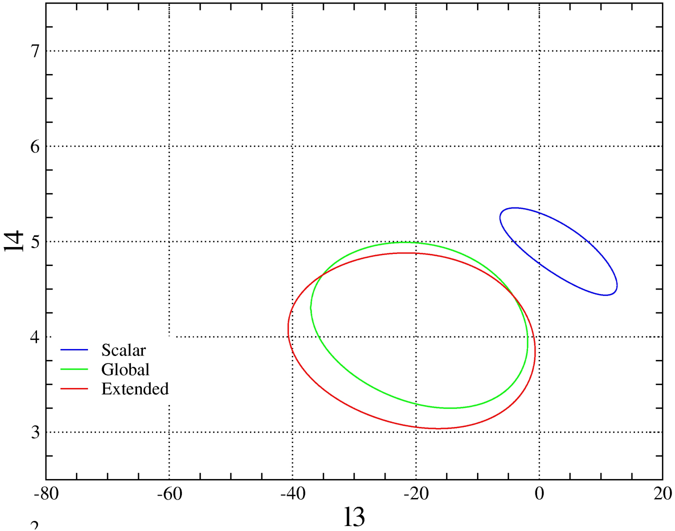
$$\begin{aligned} X(2) = 2m\Sigma(2)/(F_\pi^2 M_\pi^2) &= 1 - \delta - (4\bar{\ell}_4 - \bar{\ell}_3)\xi Y(2)^2/2 \\ Z(2) = F^2(2)/F_\pi^2 &= 1 - \varepsilon - 2\bar{\ell}_4 \xi Y(2) \end{aligned}$$

with $\xi = M_\pi^2/(16\pi^2 F_\pi^2)$ and NNLO remainders $\delta, \epsilon = O(1\%)$

(a_0^0, a_0^2): E865 data



$(\bar{\ell}_3, \bar{\ell}_4)$: E865 data

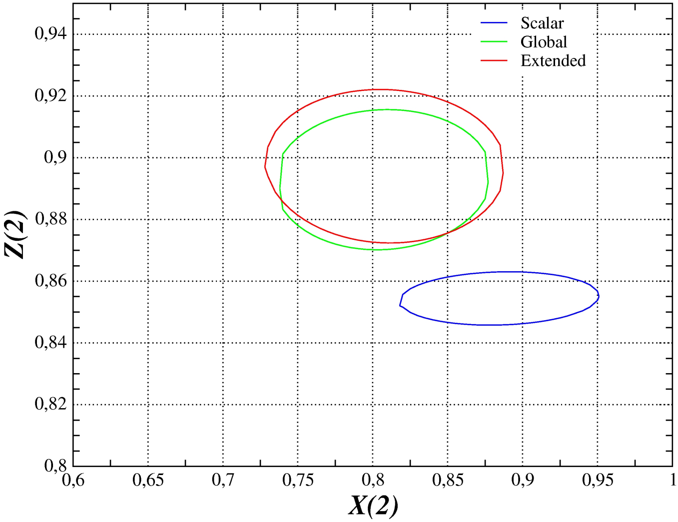


Estimates in *G.Colangelo, J.Gasser, H.Leutwyler, NPB 2001*

$$\bar{\ell}_3 = 2.9 \pm 2.4$$

$$\bar{\ell}_4 = 4.4 \pm 0.2$$

$(X(2), Z(2))$: E865 data



$$X(2) = \frac{2m\Sigma(2)}{F_\pi^2 M_\pi^2}$$

$$Z(2) = \frac{F^2(2)}{F_\pi^2}$$

Summary : E865 data

E865 only

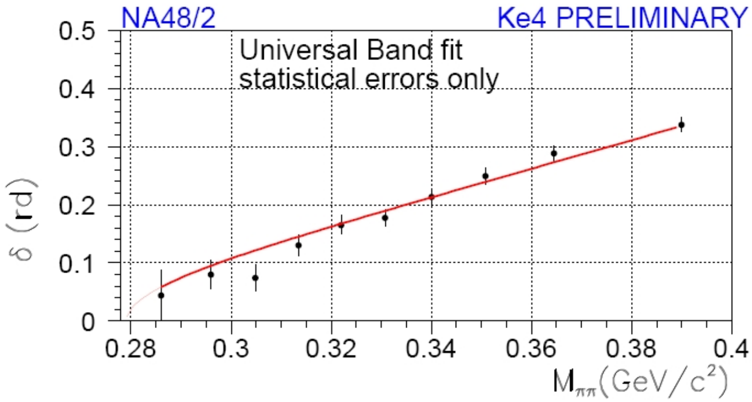
	Scalar	*Global*	*Extended*
a_0^0	0.218 ± 0.013	0.228 ± 0.012	0.228 ± 0.013
a_0^2	-0.0449 ± 0.0033	-0.0382 ± 0.0038	-0.0380 ± 0.0044
$X(2)$	0.88 ± 0.07	0.81 ± 0.07	0.81 ± 0.08
$Z(2)$	0.85 ± 0.01	0.89 ± 0.02	0.90 ± 0.03

- Disagreement between scalar and global/extended at 1 σ
- Scalar fit in agreement with expectations of standard $N_f = 2$ χPT
- Global/extended fits suggest that
  LO term in $N_f = 2$ chiral expansion of $F_\pi^2 M_\pi^2$ contributes only to 80%

Impact of NA48 data

Up to now, all results published from experimental and theoretical sides

What is the impact of NA48 preliminary data (without correlations) ?



The three fits

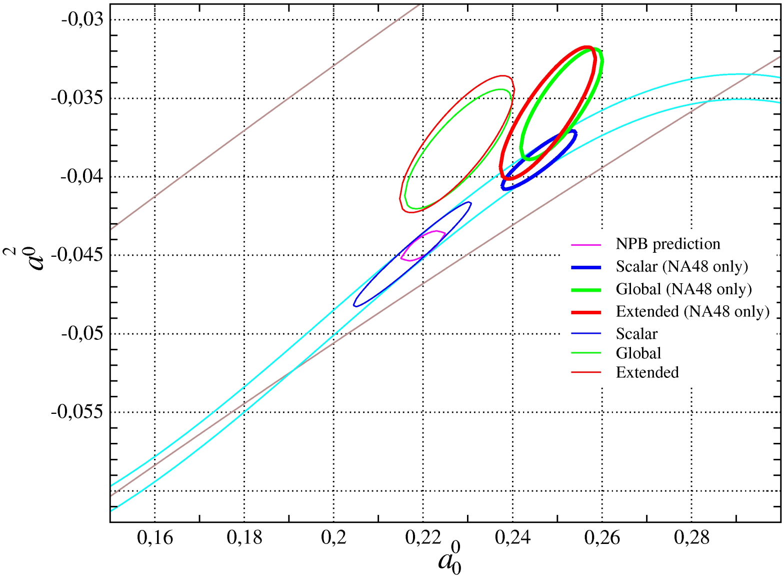
If X is a set of $K_{\ell 4}$ data

- Scalar : $\left\{ \begin{array}{l} X \\ \text{constraint from scalar radius of the pion} \end{array} \right.$
- Global : $\left\{ \begin{array}{l} (X + \text{Rosselet}) + (\text{Losty} + \text{Hoogland}) \\ \text{Roy solutions at } \theta_i \text{ fixed} \end{array} \right.$
- Extended : $\left\{ \begin{array}{l} (X + \text{Rosselet}) + (\text{Losty} + \text{Hoogland}) \\ \text{Roy solutions with } \theta_i \text{ free} \end{array} \right.$

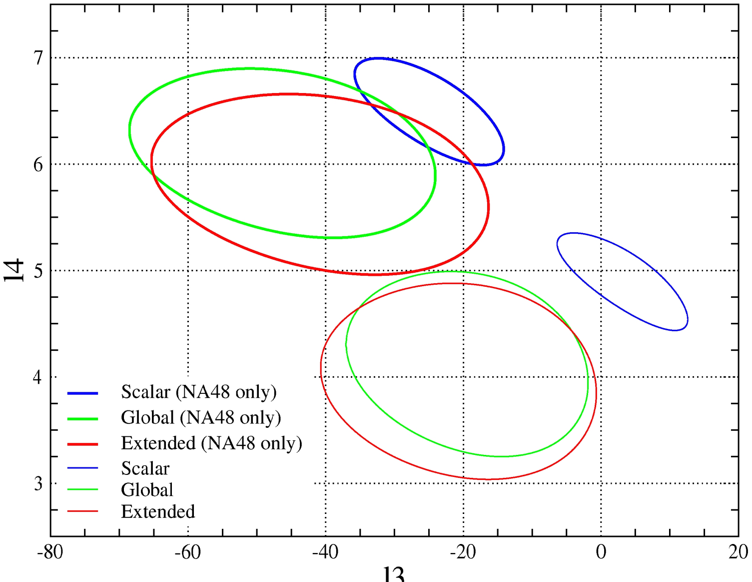
We have just seen the results for X =E865,

now, what if X=NA48 or X =NA48 + E865 ?

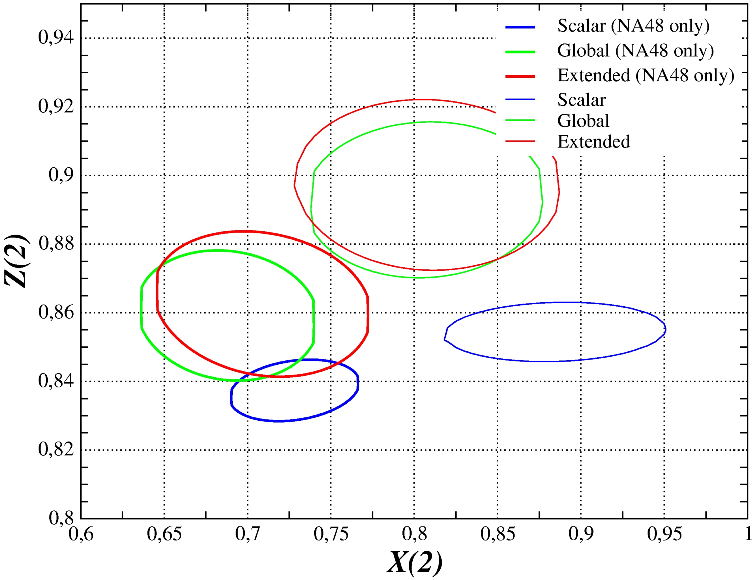
(a_0^0, a_0^2): NA48 data

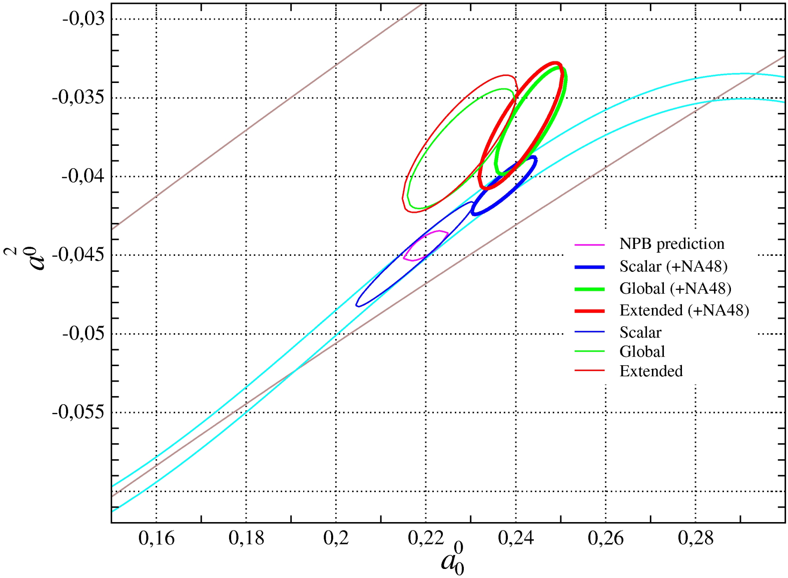


$(\bar{\ell}_3, \bar{\ell}_4)$: NA48 data

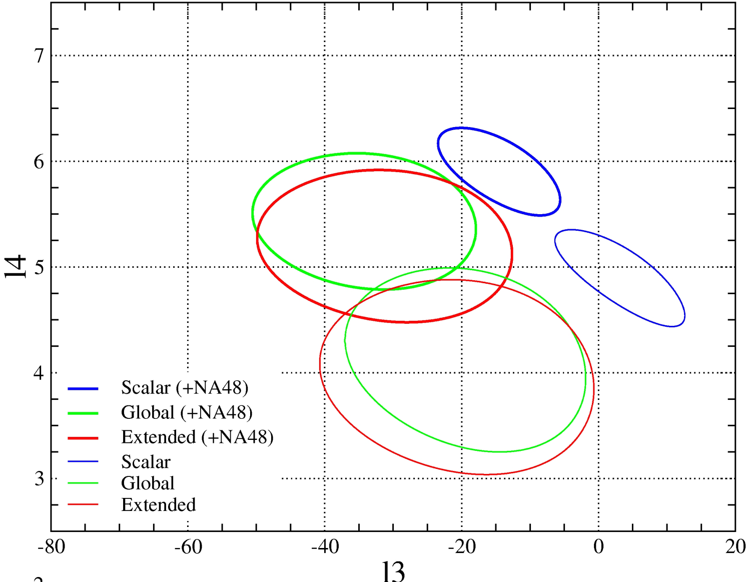


$(X(2), Z(2))$: NA48 data

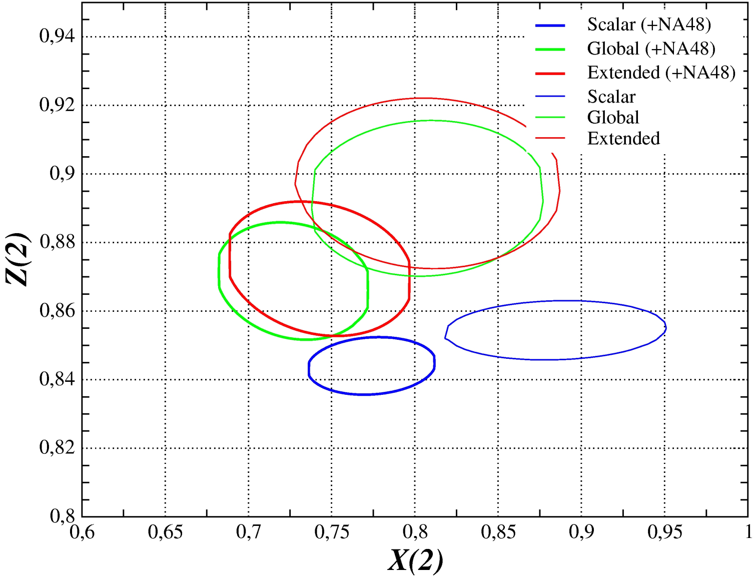



-0,03
-0,035
-0,04
-0,045
-0,05
-0,055
a_0^2
0,16
0,18
0,2
0,22
0,24
0,26
0,28
a_0^0
NPB prediction
Scalar (+NA48)
Global (+NA48)
Extended (+NA48)
Scalar
Global
Extended

$(\bar{\ell}_3, \bar{\ell}_4)$: NA48 + E865 data



$(X(2), Z(2))$: NA48 + E865 data



Summary : Impact of NA48 $K_{\ell 4}$ data

NA48 only

	Scalar	*Global*	*Extended*
a_0^0	0.246 ± 0.009	0.251 ± 0.009	0.248 ± 0.011
a_0^2	-0.0389 ± 0.0019	-0.0354 ± 0.0035	-0.0359 ± 0.0042
$X(2)$	0.73 ± 0.04	0.69 ± 0.06	0.71 ± 0.07
$Z(2)$	0.84 ± 0.01	0.86 ± 0.02	0.86 ± 0.03

NA48 + E865

	Scalar	*Global*	*Extended*
a_0^0	0.237 ± 0.008	0.243 ± 0.008	0.241 ± 0.010
a_0^2	-0.0405 ± 0.0018	-0.0365 ± 0.0034	-0.0368 ± 0.0040
$X(2)$	0.77 ± 0.04	0.73 ± 0.05	0.74 ± 0.06
$Z(2)$	0.84 ± 0.01	0.87 ± 0.02	0.87 ± 0.03

- Improved agreement between scalar and global/extended
- $N_f = 2$ LO contribution to $F_\pi^2 M_\pi^2$ around 70%, and to F_π^2 around 85%

Impact of the cusp in $K \to 3\pi$

$K \to 3\pi$ cusp related to $\pi^+\pi^- \to \pi^0\pi^0$ and proposed as an alternative to extract scattering lengths

N. Cabibbo; N.Cabibbo and G.Isidori

G.Colangelo, J.Gasser, B.Kubis, A.Rusetsky

and studied by the NA48 collaboration

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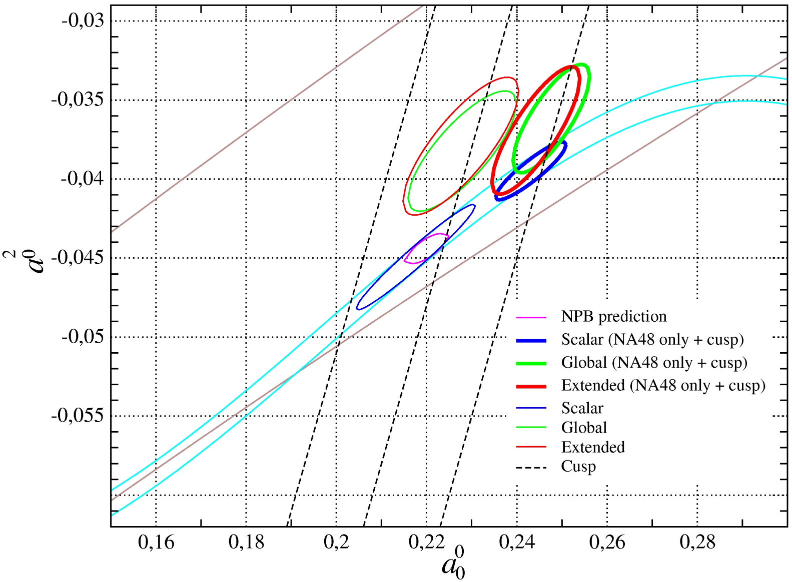
Same fits as before, but

- Take only NA48 for $K_{\ell 4}$ (neither Rosselet nor E865 included)
- Include the NA48 results from the cusp (hep-ex/0511056)

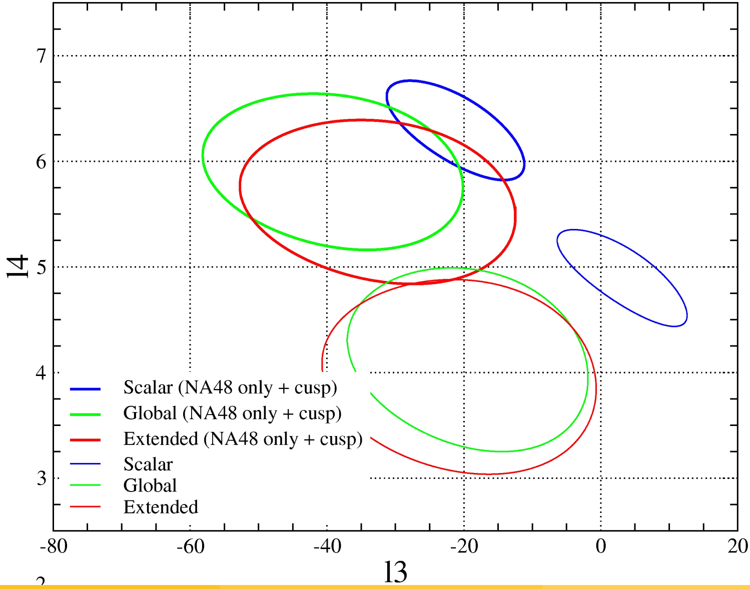
$$a_0^0 - a_0^2 = 0.268 \pm 0.017 \qquad a_0^2 = -0.041 \pm 0.026$$

adding errors quadratically and neglecting correlations in stat errors

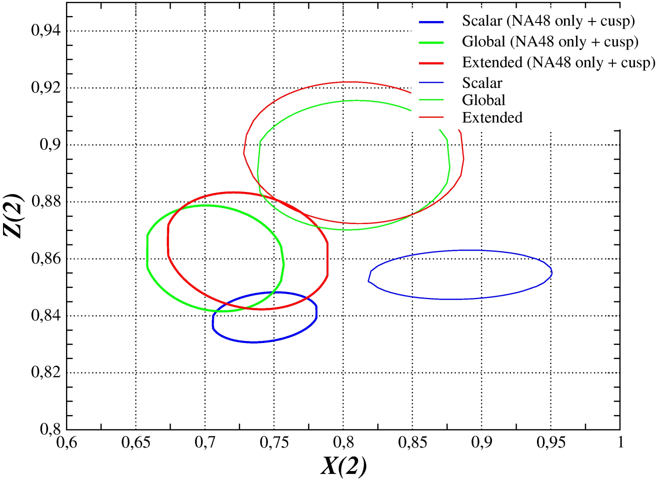
(a_0^0, a_0^2): NA48 $K_{\ell 4}$ and cusp



(ℓ_3, ℓ_4): NA48 $K_{\ell 4}$ and cusp

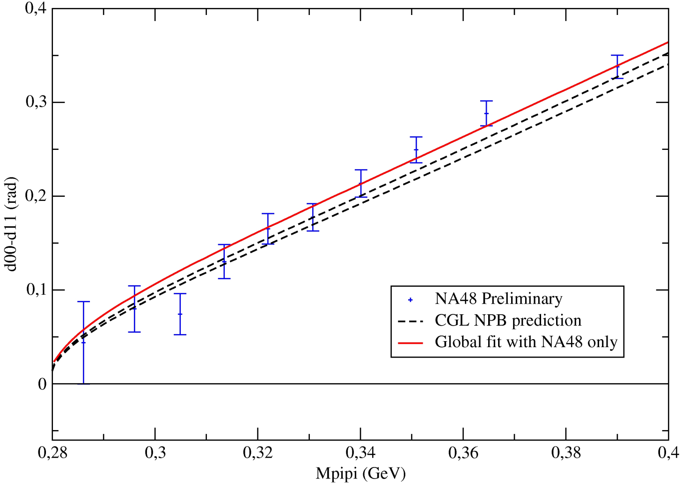


$(X(2), Z(2))$: NA48 $K_{\ell 4}$ and cusp



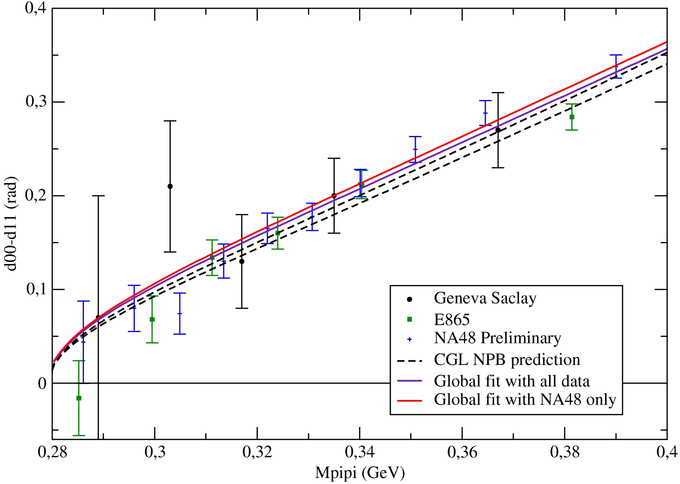
Situation very close to previous fits without cusp

Phase shifts



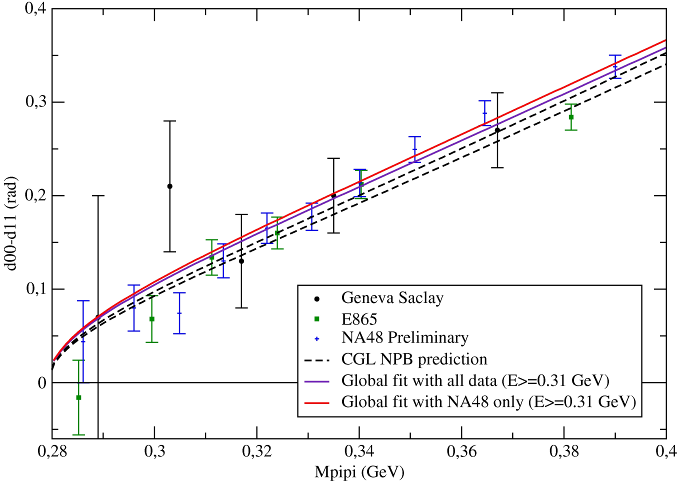
In red, minimum of χ^2 for the global fit with NA48 alone

Threshold versus higher energy (1)



What is the impact
of the first points near threshold ?

Threshold versus higher energy (2)



Same fit with only points above 0.31 GeV and almost same result

$$a_0^0 = (0.251 \to 0.247) \pm 0.009 \qquad a_0^2 = (-0.0354 \to -0.0352) \pm 0.0035$$

Conclusions

- $K_{\ell 4}$ data useful to determine $\pi\pi$ scattering lengths and open a window on QCD dynamics in $N_f = 2$ chiral limit
- Unfortunately, need to add more information
  - scalar radius of the pion (scalar fit)
  - $I = 2$ data (global/extended fit)
- Recent E865 $K_{\ell 4}$ data indicated a disagreement between the fits
  - scalar fit agreed with expectations of standard $N_f = 2$ χPT
  - but not global/extended (discrepancy at 1 σ)

Conclusions (2)

- Preliminary NA48 $K_{\ell 4}$ data shift the scattering lengths leading to a better agreement among the fits
- But they imply a slow convergence of $N_f = 2$ chiral expansions

$$X(2) = \frac{LO[F_\pi^2 M_\pi^2]}{F_\pi^2 M_\pi^2} \simeq 0.70 \qquad Z(2) = \frac{LO[F_\pi^2]}{F_\pi^2} \simeq 0.85$$

- NA48 results on the $K \to 3\pi$ cusp do not alter significantly the results for scattering lengths and $N_f = 2$ order parameters
- Fits seem more sensitive to the slope of the phase shifts at high energy than to the threhsold data

If results confirmed, significant impact
on the convergence of $N_f = 2$ and $N_f = 3$ chiral expansions