How to determine a_0^0 and a_0^2 from K_{e4} data

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Outline

The $\pi\pi$ scattering amplitude

- Scattering amplitude of two pions in a fixed isospin state

$$T^I(s,t) \qquad I = 0,1,2$$

- Partial wave decomposition

$$T^I(s,t) = 32\pi \sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

- Phase shift and elasticity

$$t_\ell^I(s) = \frac{1}{2i\sigma(s)} \left\{ \eta_\ell^I(s) e^{2i\delta_\ell^I(s)} - 1 \right\} \qquad \sigma(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

The inelastic threshold is at $s = 16M_\pi^2$ in principle.
In practice, it is higher, *e.g.* $4M_K^2$ in the $I = 0$ S wave.

Why use Roy equations in the data analysis?

- in $\pi\pi$ scattering the process is the same in all channels: crossing symmetry relates amplitudes with different isospin;
- Roy equations are dispersion relations which incorporate (partly) crossing symmetry as well as analyticity and unitarity;
- all partial waves are related to each other and at low energy they all essentially depend on only two parameters:

$$a_0^0 \text{ and } a_0^2$$

- analyzing any low-energy experiment on $\pi\pi$ scattering by means of the Roy equations allows the translation of very different data sets into values of the two S wave scattering lengths

Why use Roy equations in the data analysis?

Disadvantages:

- the solution of the Roy equations is not known analytically: the dependence of the phase shifts on a_0^0 and a_0^2 is implicit;
- at best it can be parametrized numerically in the form of a parametrization (cf. ACGL(01) and DFGS(02));

Roy solutions

- given a set of "high-energy" input parameters and the two *S*-wave scattering lengths the Roy equations fix uniquely the phase shifts at low energy
- the two most important input parameters are the phases of the *S*0 and *P* wave at 0.8 GeV:

$$\delta_0^0(0.8) \quad \text{and} \quad \delta_1^1(0.8)$$

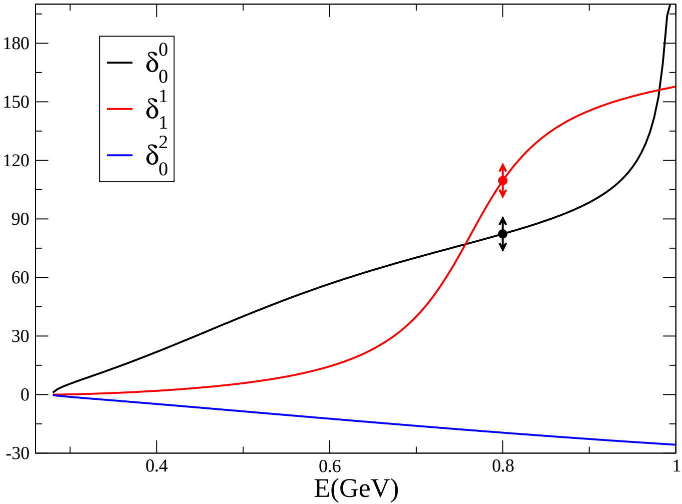
- data on $e^+e^- \to \pi^+\pi^-$ pin down the *P* wave very accurately

$$\delta_1^1(0.8) = (109.6 \pm 1)^\circ$$

- the *S*0 phase is more uncertain

$$\delta_0^0(0.8) = 82.3^\circ \; {}^{+10^\circ}_{-4^\circ}$$

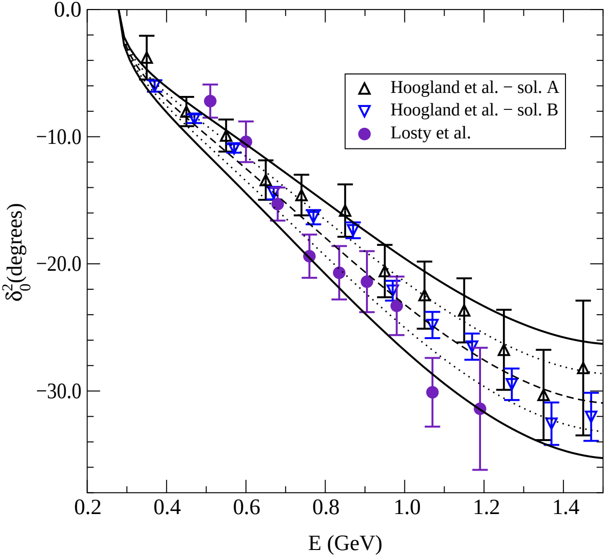
Roy solutions



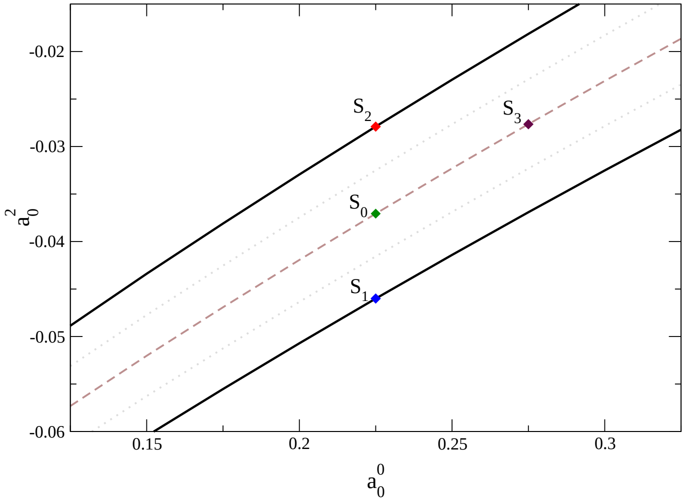
Universal band

- In the $I = 2$ S wave there is no freedom in choosing $\delta_0^2(0.8)$: once a_0^2 is fixed, the Roy equations admit a physical solution only for a single $\delta_0^2(0.8)$;
- $\Rightarrow$ data on the $I = 2$ S wave phase shift translate into a range of values for a_0^2;
- the relation $a_0^2 \Leftrightarrow \delta_0^2(0.8)$ depends on the value of a_0^0: data on the $I = 2$ S wave phase shift translate into a band in the (a_0^0, a_0^2)–plane, called the Universal Band;

Universal band



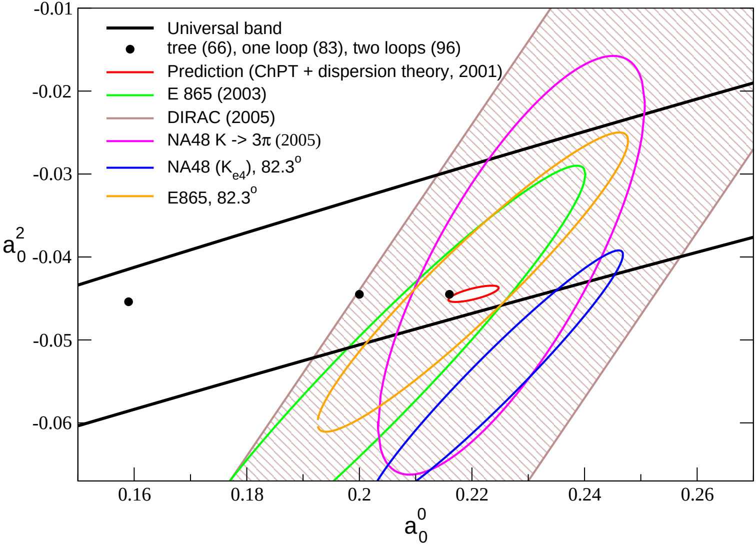
Universal band



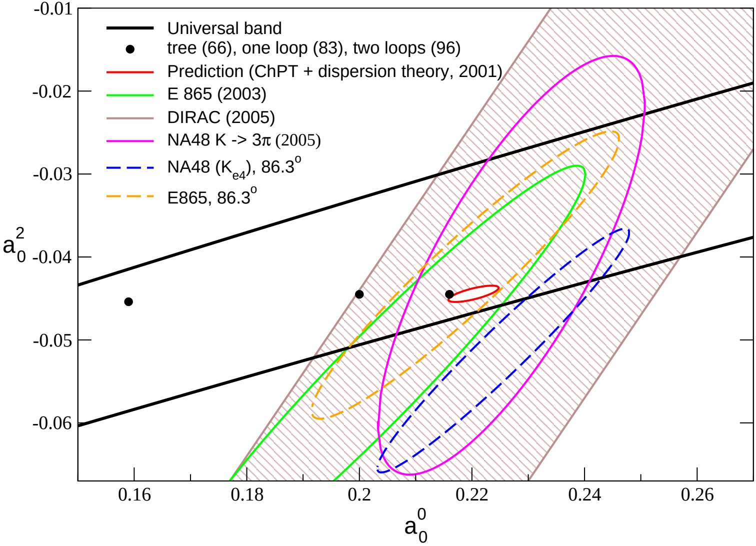
How to analyze the data?

- for a fixed "high-energy" input, to every point in the (a_0^0, a_0^2) plane corresponds a unique set of phase shifts;
- given a data set on some phase shifts (*e.g.* the K_{e4} data on $\delta_0^0 - \delta_1^1$) to every point in the (a_0^0, a_0^2) plane corresponds a χ^2 value;
- as usual, one can calculate χ^2 minima and one-sigma contours, for a given set of data or a combination of different data sets;
- the special status of the $I = 2$ S wave makes the corresponding data be immediately visible in the (a_0^0, a_0^2) plane in the form of the Universal Band;
- on the other hand these data are not privileged with respect to others – if another data set prefers the region outside the Universal Band, this means a clash between two data sets, not that the latter contradicts the Roy equations

1 σ ellipses



1 σ ellipses



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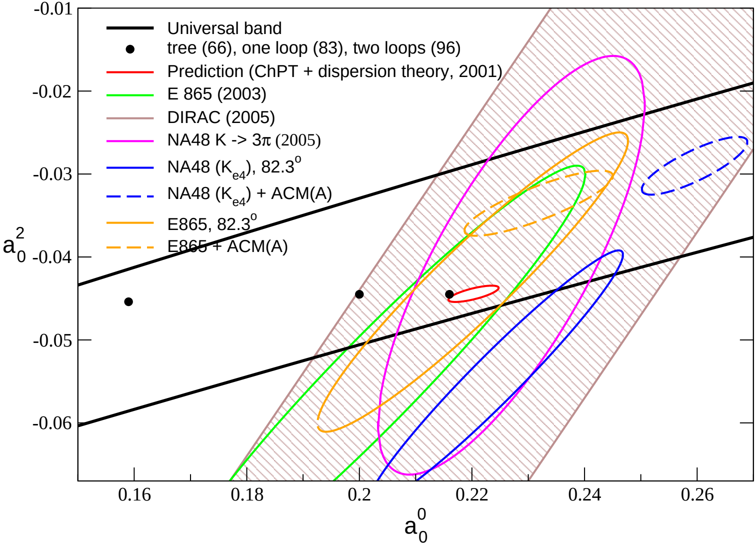
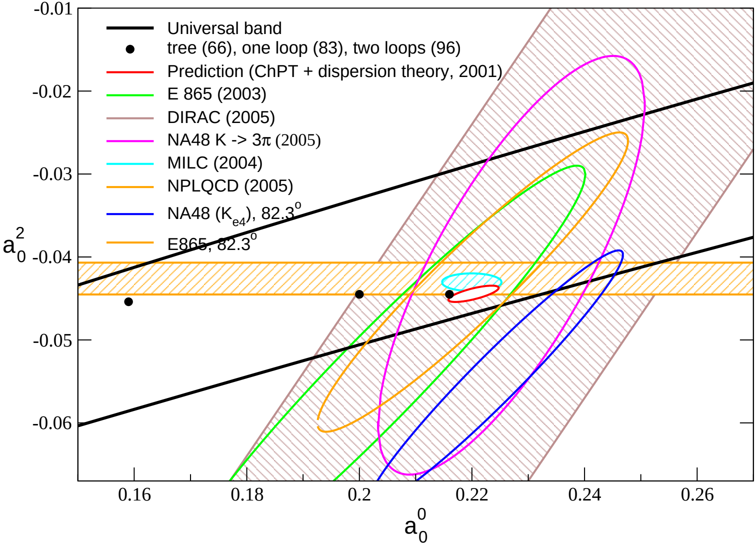


1 σ ellipses



Conclusions

- The E865 and NA48 data sets on K_{e4} decays both allow a precise determination of the two S-wave scattering lengths. The two determinations, however, are somewhat in disagreement
- Minima of the χ^2:

$$\begin{array}{rlll} \text{E865:} & a_0^0 = 0.220, & a_0^2 = -0.0430, & \chi^2_{\rm min} = 6.2 \\ \text{NA48:} & a_0^0 = 0.224, & a_0^2 = -0.0546, & \chi^2_{\rm min} = 6.0 \\ \text{NA48+E865:} & a_0^0 = 0.226, & a_0^2 = -0.0491, & \chi^2_{\rm min} = 16.1 \end{array}$$

- The numbers given here are preliminary and may change. The problems we have pointed out will have to be sorted out.