NA48/2 -Ke4 analysis status

Outline

NA48/2: Data statistics, event selection

Ke4 formalism : form factors and phase

Preliminary results (Summer conferences 06)

More on Systematics uncertainties

Short term perspectives

Summary

Data statistics

Study of Ke4 rare decays in the "charged" $\pi^+\pi^- e^\pm \nu$ and "neutral" $\pi^0\pi^0 e^\pm \nu$ final states, both modes with small BR's of few 10^{-5}

2003 Run ~50 days

2004 Run ~60 days

Total statistics :

~4. 10^9 $\pi^+\pi^-\pi^\pm$ decays and ~1. 10^8 $\pi^0\pi^0\pi^\pm$ decays

~1. 10^6 $\pi^+\pi^- e^\pm \nu$ decays and ~3.7 10^4 $\pi^0\pi^0 e^\pm \nu$ decays

Preliminary results (presented in Summer Conferences 2006):

charged K_{e4} based on 370000 charged decays (30 days in 2003)

neutral K_{e4} based on 2003 statistic for Br (~10000 events) and full (2003+2004) for form factors (~30000 events).

Charged Ke4 : SS123 = 2003 partial statistics

Partial statistics from 2003 (30 days/50 days) available with Ke4 selection and background (~0.5%) subtracted

Data selected	K+	K-	all
S123	236839	131848	368687 (Conf. result)

MC generated (for 2 x 112 runs = different beam/detector conditions)

accepted events	K+	K-	total
SS123	5.5 Millions	3.0 Millions	8.5 Millions

Ratio K+/K- ~ 1.8 both in Data and MC (run by run basis)
Ratio MC/Data ~ 23. both for K+ and K- (run by run basis)

Ke4 charged decays : event selection

Signal $\pi^+\pi^-e^{\pm}\nu$ **Topology** :

- 3 charged tracks with a "good" vertex,
- two opposite sign pions,
- one electron (LKr info E/p),
- some missing energy and p_T (neutrino)

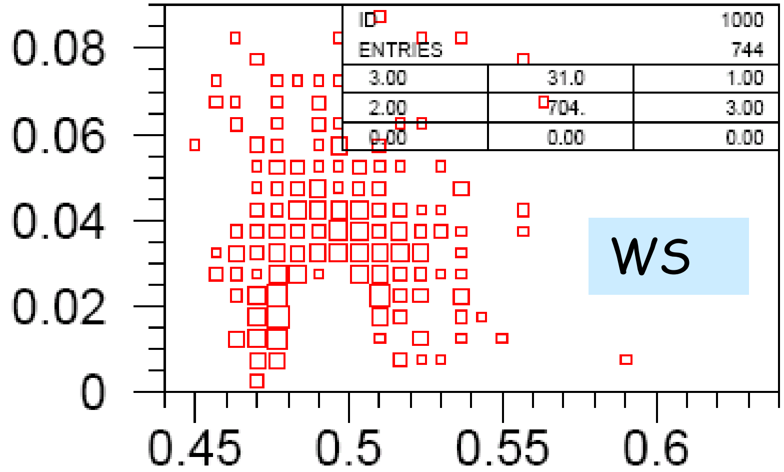
Background : main sources

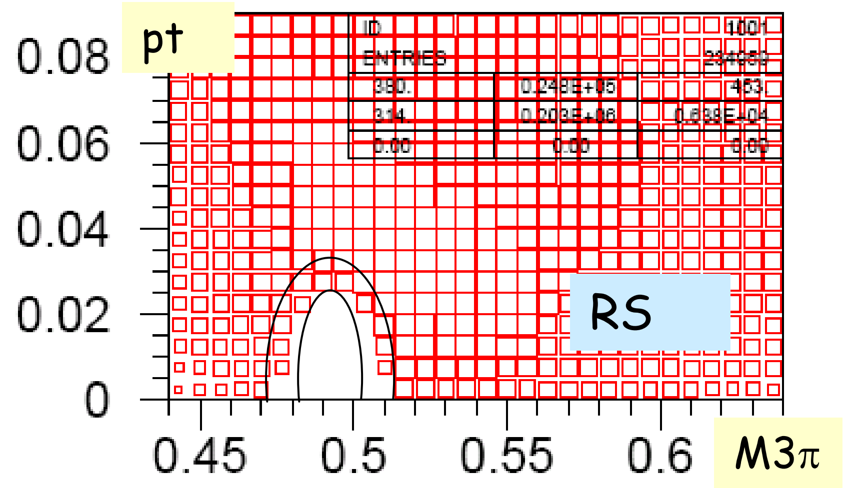
$\pi^{\pm}\pi^+\pi^-$ decay + $\pi \to e\,\nu$ decay (dominates with same topology as signal)
+ π misidentified as e

$\pi^{\pm}\pi^0(\pi^0)$ decay + π^0 Dalitz decay ($e^+e^-\gamma$) with e misidentified as π and γ (s) undetected

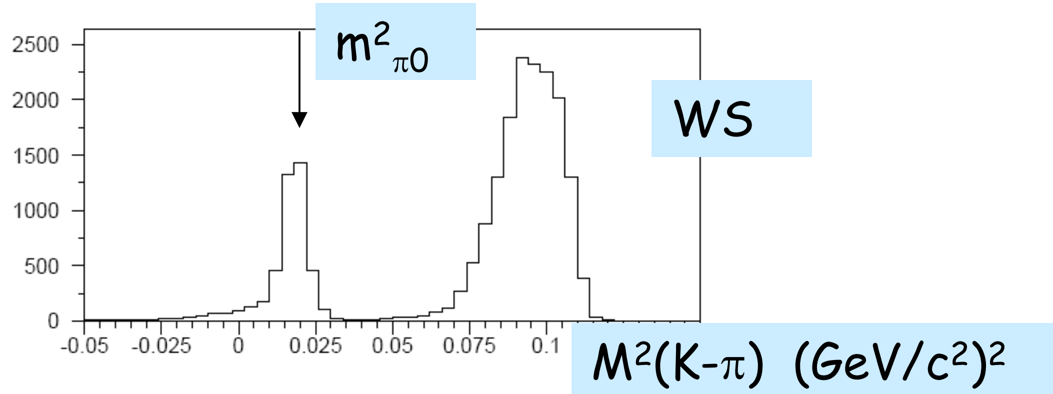
Control from data sample : Wrong Sign events have the same total charge as selected events but same sign pions. Depending on the background process, events appear in Right Sign events with the same rate ($\pi^{\pm}\pi^0(\pi^0)$) as in WS events or twice the rate ($\pi^{\pm}\pi^+\pi^-$)

- Against $\pi^{\pm}\,\pi^{+}\,\pi^{-}$: elliptic cut in the plane ($m_{3\pi}$, p_T) assigning m_{π} to each particle (loose, medium and tight ellipses, only a few percents signal loss)





- Against $\pi^{\pm}\pi^{0}_{D}$: missing mass to (K-$\pi^{\pm}$) larger than $m_{\pi 0}$



Ke4 charged decays : background rejection

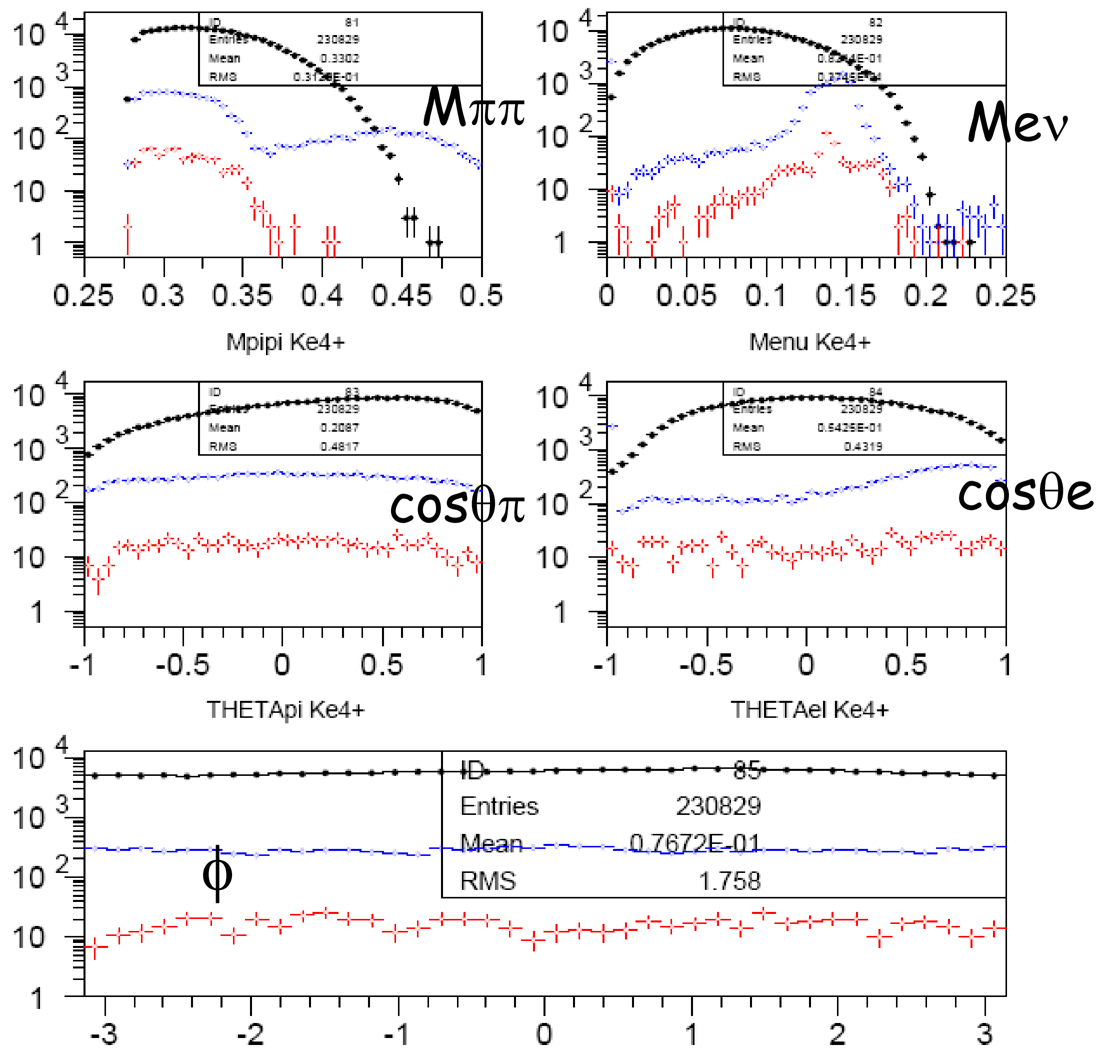
Additional e-π rejection from Linear Discrimant Analysis (LDA) or Neural network (NN) methods using shower shape variables.

Black : Data Right sign

Blue : data Wrong Sign (loose selection)

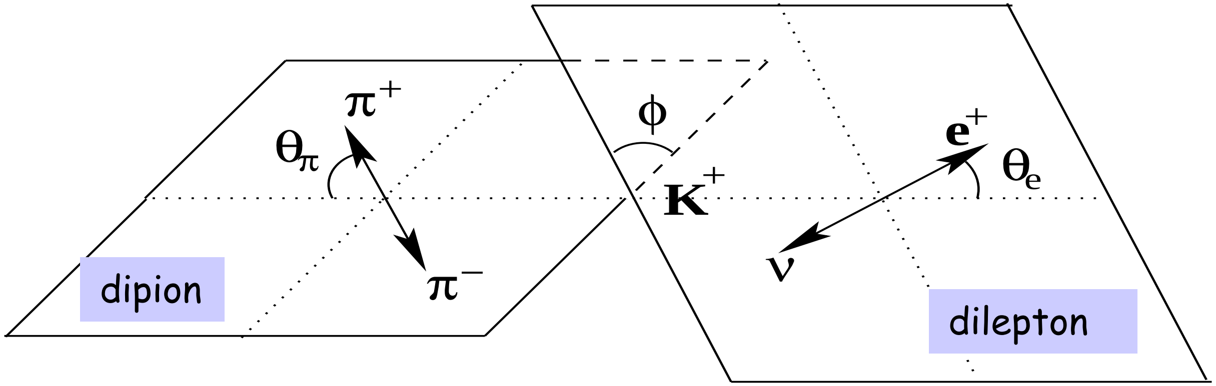
Red : data Wrong Sign (tight selection)

Total background level can be kept at <1% relative level



Ke4 charged decays : formalism

The Ke4 decay is described using 5 kinematic variables (as defined by Cabibbo-Maksymowicz): S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$), $\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



The form factors which appear in the decay rate can be measured from a fit to the experimental data distribution of the 5 variables provided the binning is small enough.

Several formulations of the form factors appear in the literature, we have considered two of them, proposed by Pais and Treiman (Phys.Rev. 168 (1968)) and Amoros and Bijnens (J.Phys. G25 (1999)) which can be related.

Ke4 charged decays : formalism

Using a partial wave expansion (S,P,D …):

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi + $ D-wave term…

$G = G_p e^{i\delta g} +$ D-wave term…

$H = H_p e^{i\delta h} +$ D-wave term…

Keeping only S and P waves (S_π is small in Ke4) , rotating phases by δ_p and assuming $(\delta_g - \delta_p) = 0$ and $(\delta_h - \delta_p) = 0$, only 5 form factors are left:

$F_s \quad F_p \quad G_p \quad H_p$ and $\delta = \delta_s - \delta_p$

developing in powers of q^2 ($q^2 = (S_\pi/4m_\pi^2) - 1$), $S_e/4m_\pi^2$ …

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e \left(S_e/4m_\pi^2\right) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

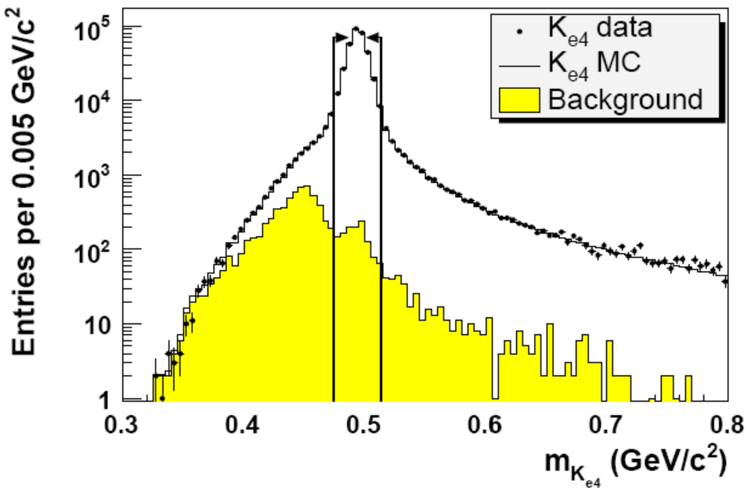
$$H_p = h_p + h_p' q^2 + ..$$

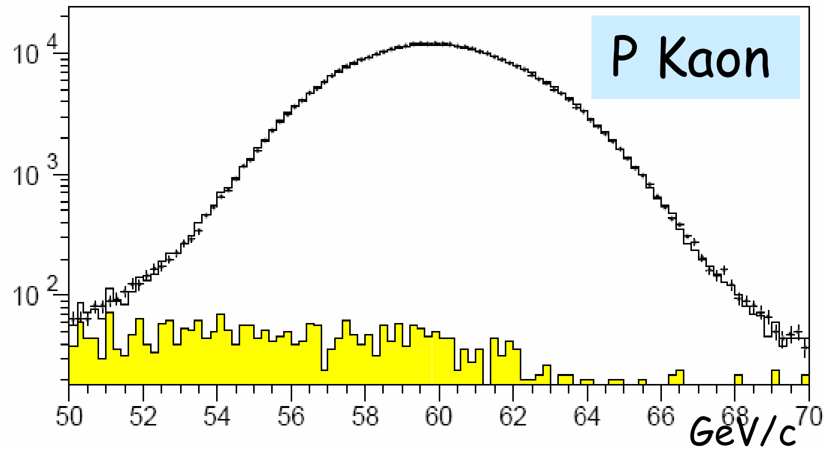
Reconstruction of the C.M. variables : Two options

• impose the Kaon mass, use ν constrain to solve energy-momentum conservation equations and get P_K

• impose a 60 GeV/c Kaon momentum, assign the missing p_T to the ν and compute the mass of the system (π π e ν)

Then boost particles to the Kaon rest frame and dipion/dilepton rest frames to get the angular variables.





Using **equal population bins** in the 5-dimension space of the C.M. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) one defines a grid of **10x5x5x5x12=15000 boxes**.

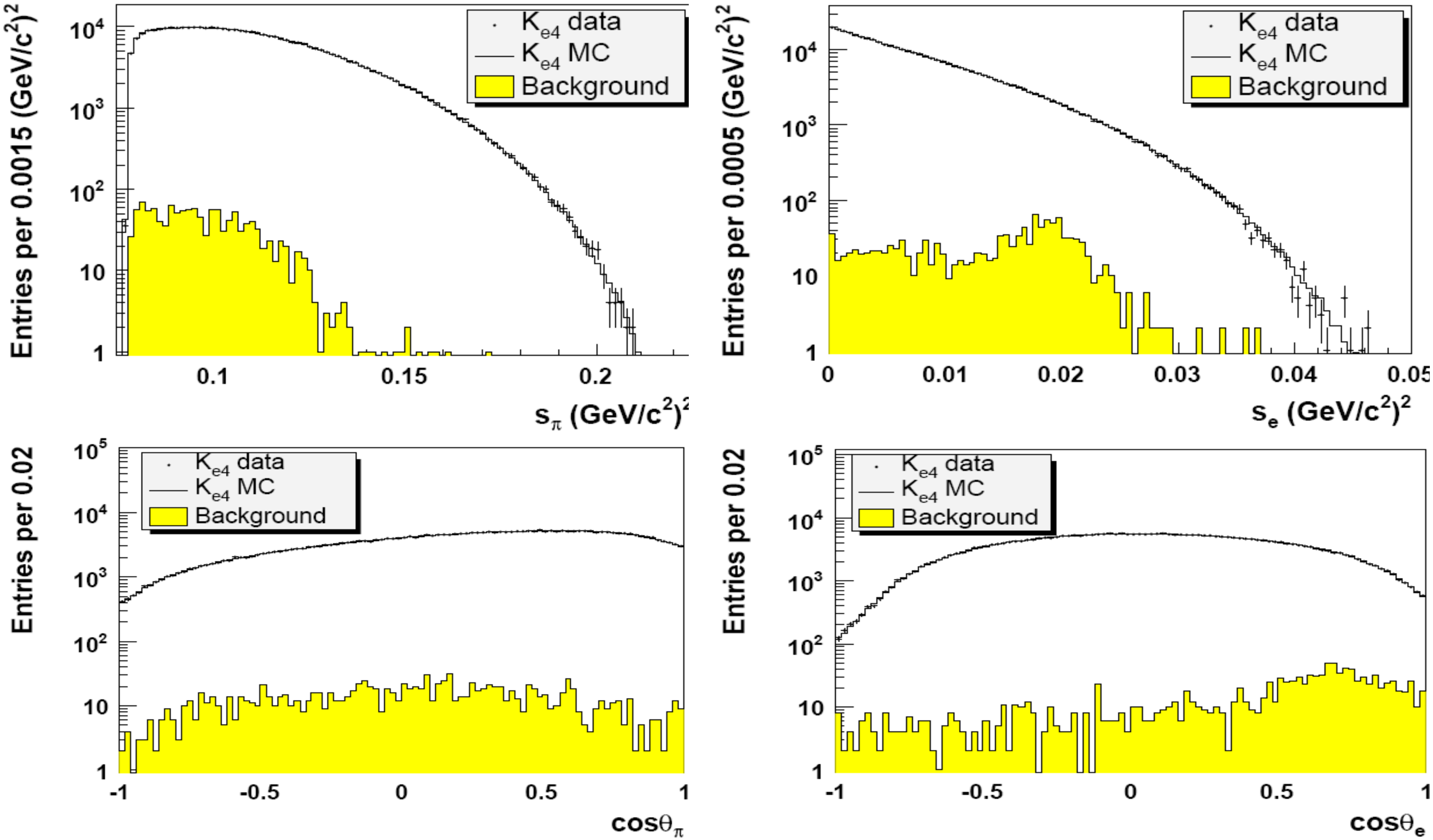
The set of form factor values are used to minimize the T^2, a log-likelihood estimator well suited for small numbers of data events/bin Nj and taking into account the statistics of the simulation = Mj simulated events/bin and Rj expected events/bin.

$$T^2 = 2\sum_{J=1}^{S} \{ N_j Log\left[\frac{N_j}{R_j}(1-\frac{1}{Mj+1})\right] + (N_j + M_j + 1) Log\left[\frac{1+\frac{R_j}{M_j}}{1+\frac{N_j}{M_j+1}}\right] \}$$

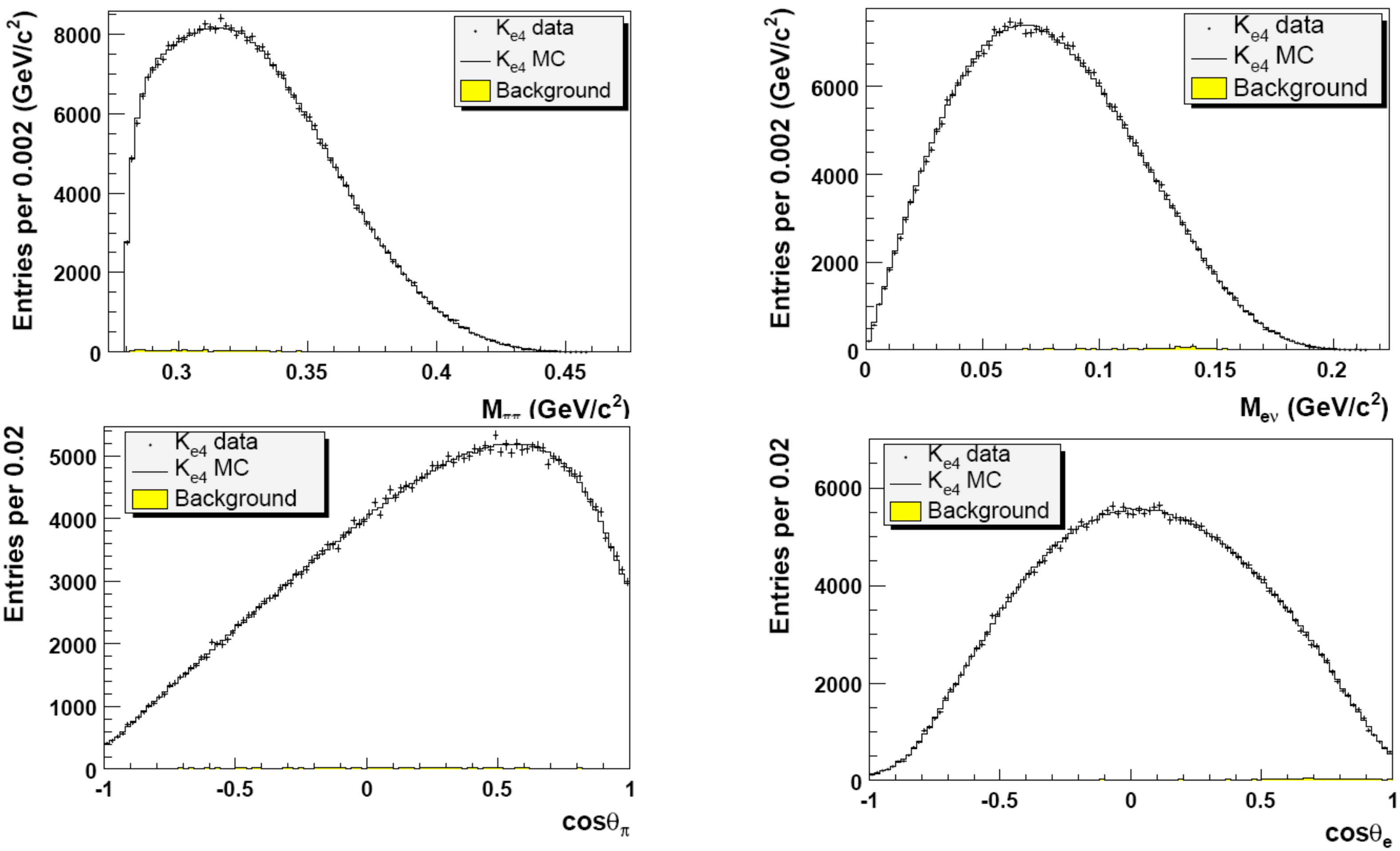
For the K^+ sample (235000 events), there are 16 events/bin

For the K^- sample (132000 events), there are 9 events/bin

Ke4 charged decays : 4 C.M. distributions (log scale)

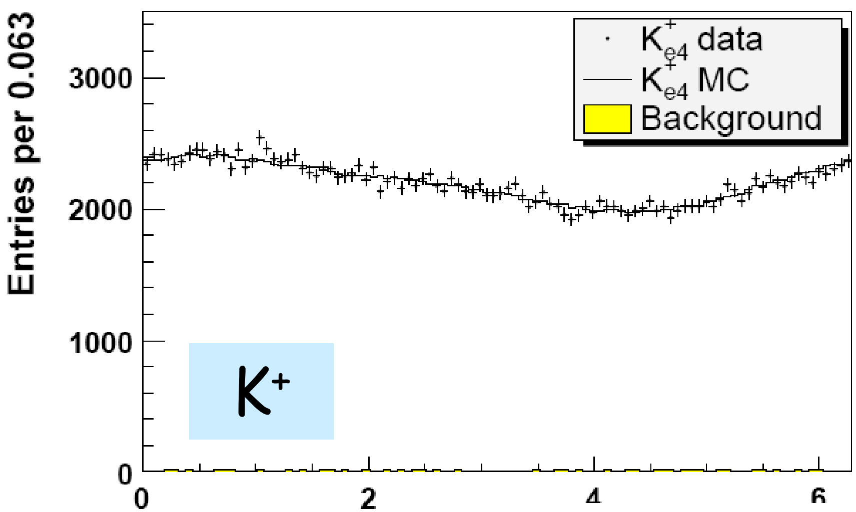


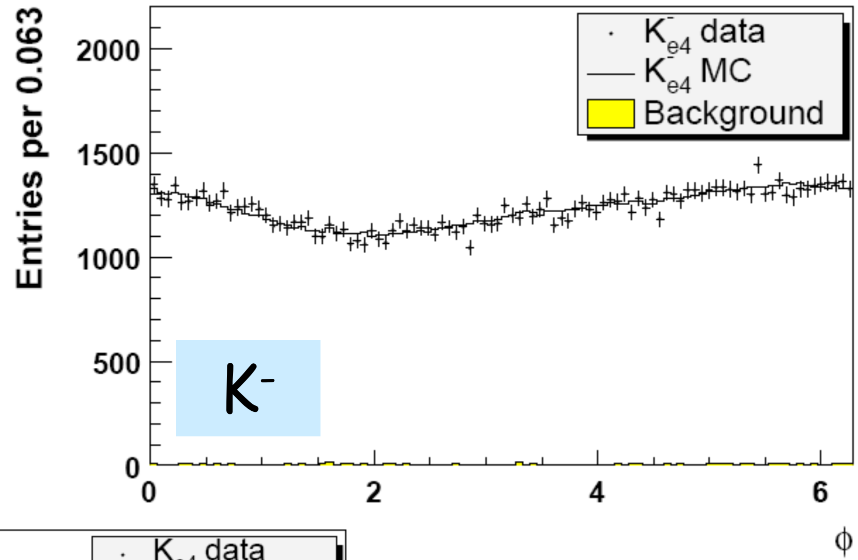
Ke4 charged decays : 4 C.M. distributions (Lin. Scale)

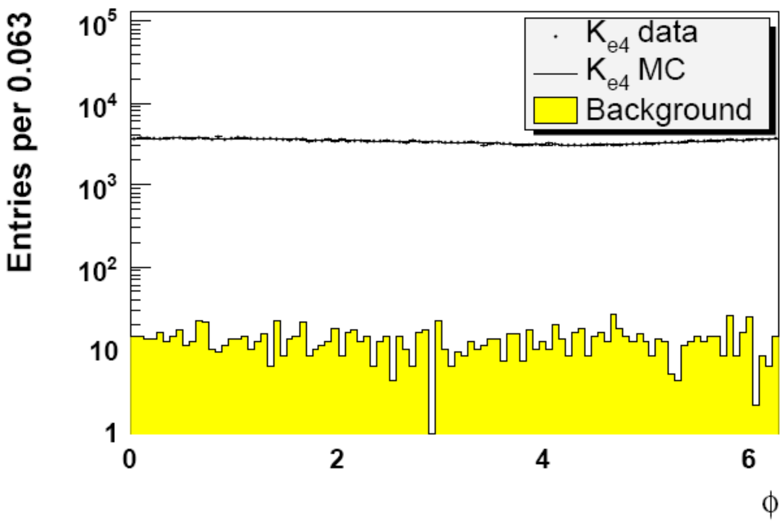


Ke4 charged decays : the 5th distribution ϕ

CP symmetry : (K^{+}) ϕ distribution is opposite of (K^{-}) ϕ distribution







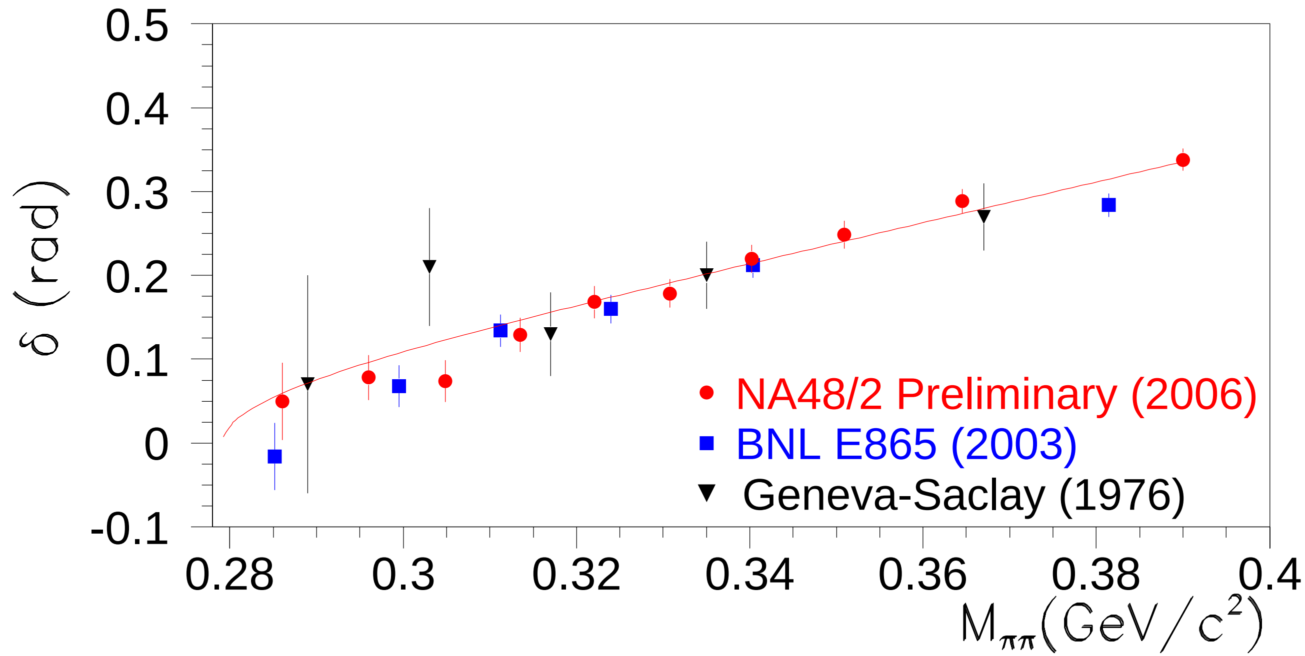
$K^{+}(\phi) + K^{-}(-\phi)$

Note the log scale to see the background!

Ke4 charged decays : form factors

- **Ten independent fits**, one in each $M_{\pi\pi}$ bin, assuming ~constant form factors over each bin. This allows a model independent analysis.
- **Use** a parameterization to extract a_0^0 with a fixed relation $a_0^2 = f(a_0^0)$ (ie Roy equations to extrapolate to low energy and constrain to the middle of the Universal Band) (**ACGL** Phys. Rep.353 (2001), **DFGS** EPJ C24 (2002))

$$a_0^2 = -0.0849 + 0.232 a_0^0 - 0.0865 \left(a_0^0\right)^2 \quad [\pm 0.0088]$$

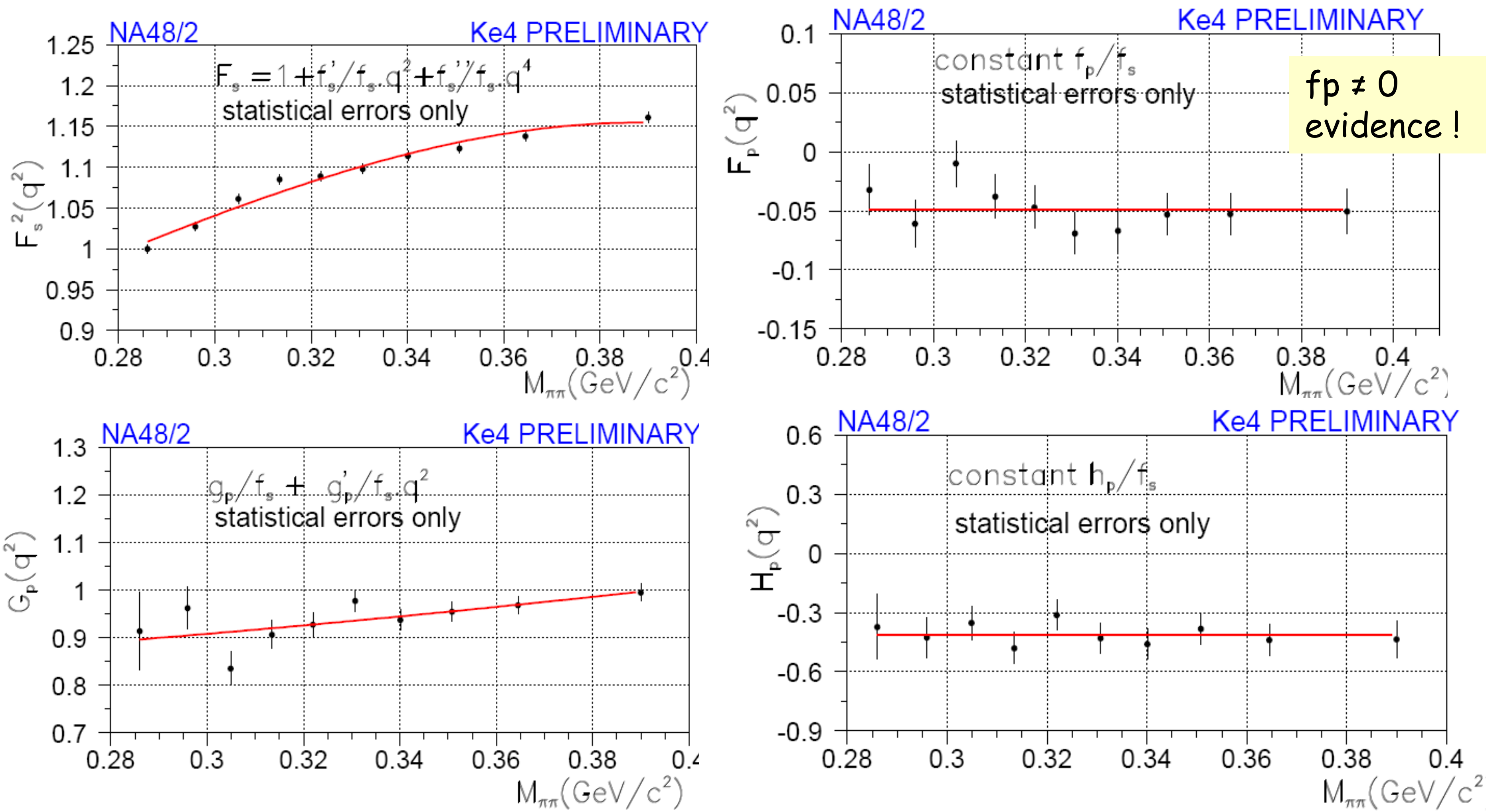


Summer conferences results

K^+ and K^-	result	statistical error	systematic error
f_s' / f_s	0.169	$\pm$ 0.009	$\pm$ 0.034
f_s'' / f_s	-0.091	$\pm$ 0.009	$\pm$ 0.031
f_p / f_s	-0.047	$\pm$ 0.006	$\pm$ 0.008
g_p / f_s	0.891	$\pm$ 0.019	$\pm$ 0.020
g_p' / f_s	0.111	$\pm$ 0.031	$\pm$ 0.032
h_p / f_s	-0.411	$\pm$ 0.027	$\pm$ 0.038
a_0^0 (UB) implying a_0^2(UB)	0.258 -0.031	$\pm$ 0.008 $\pm$ 0.002	$\pm$ 0.007 $\pm$ 0.018 Theory (UB width) $\pm$ 0.002 $\pm$ 0.009 Theory (UB width)

Ke4 charged decays : form factors

Relative form factors and their variations with q2,q4 (Se dependence consistent with 0)

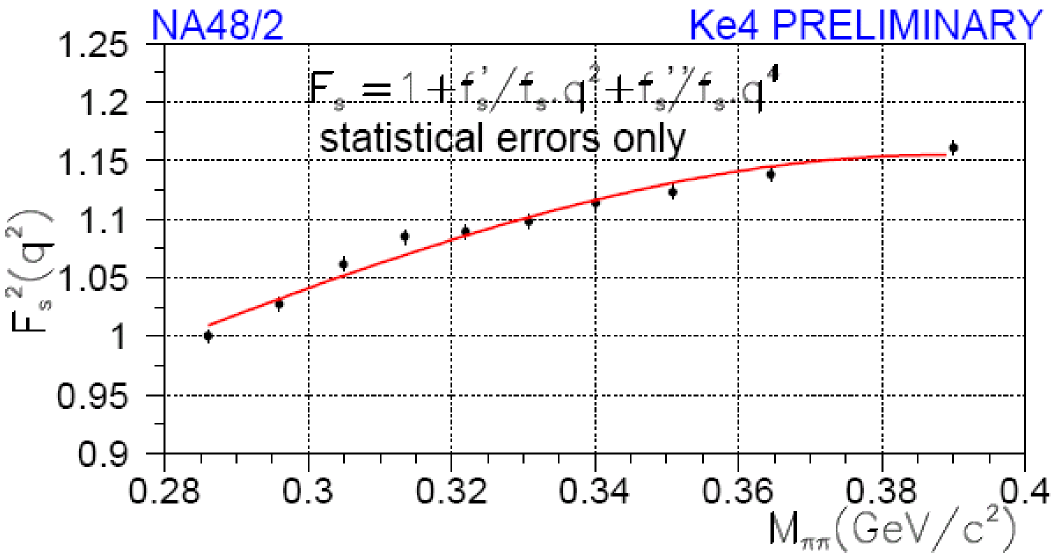


Investigation of Se dependence for Fs

The q2 dependence of Fs was measured through the variation of the normalization Ndata/Nmc(fit) per bin, proportional to Fs^2.

$$Fs = fs0(1. + fs'/fs0\, q^2 + fs''/fs0\, q^4 + fe/fs0(Se/4m^2\pi))$$

$$q^2 = (S\pi / 4m^2\pi - 1), \quad Se = M^2 e\nu$$



The fe term was not considered in this approach.

To investigate a possible Se dependence, the normalization was studied also as a function of $Me\nu$ and a two-dimension distribution was fitted

Investigation of Se dependence for fs

NEW

K+ and K- combined (results are similar for each sign alone)

3parameter fit (f',f'',norm)

$f'/fs0 = 0.1711 \pm 0.0121$

$f''/fs0 = -0.0949 \pm 0.0120$

$\chi 2/ndf = 1.90$ (ndf=87-3)

$\rho\ (f',f'') = -0.96$

4parameter fit (f',f'',fe,norm)

$f'/fs0 = 0.1773 \pm 0.0128$

$f''/fs0 = -0.0923 \pm 0.0126$

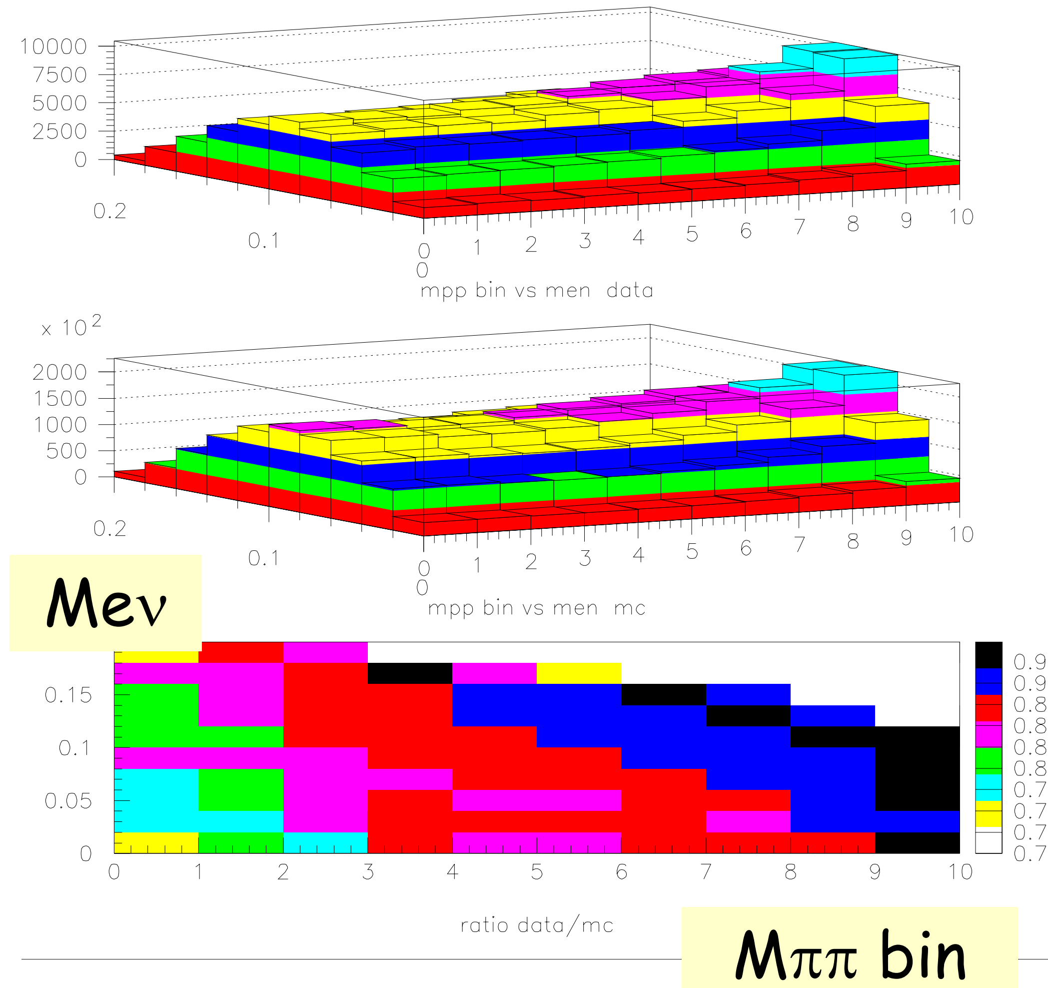
$fe/fs0 = 0.0811 \pm 0.0109$

$\chi 2/ndf = 1.27$ (ndf=87-4)

$\rho(f',f'') = -0.96$

$\rho(f',fe) = 0.03$

$\rho(f'',fe) = -0.06$



Ke4 neutral decays

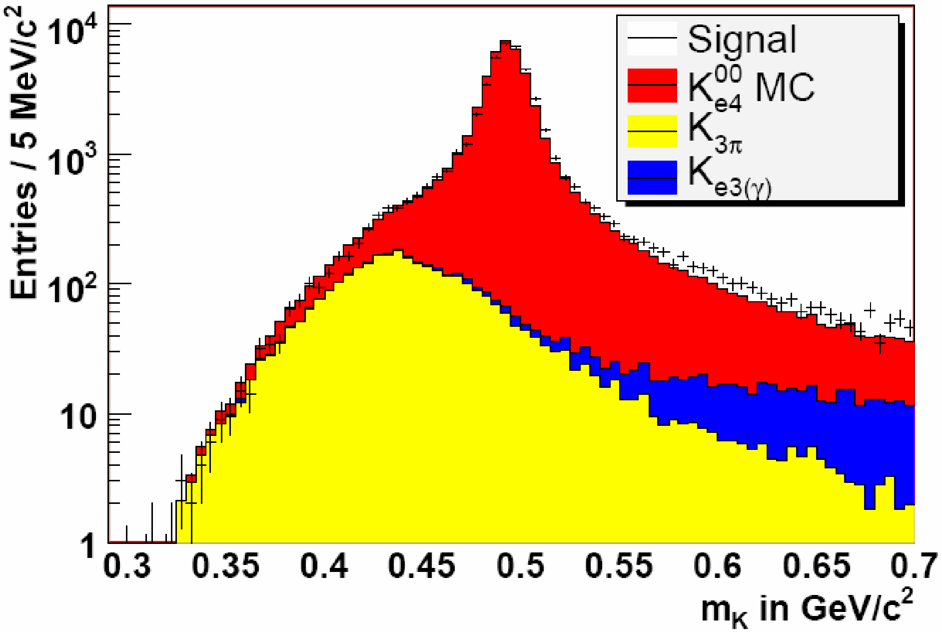
Signal $\pi^0\pi^0 e^{\pm}\nu$ **Topology** : 1 charged track , 2 π^0s (reconstructed from **4**γ**'s** in LKr), 1 electron (LKr info E/p and shower width), some missing energy and p_T (neutrino)

Background : main sources

$\pi^{\pm}\,\pi^0\,\pi^0$ decay + π misidentified as e (dominant)

$\pi^0\,e^{\pm}\,\nu\,\gamma$ decay + accidental γ

Total contamination ~3%

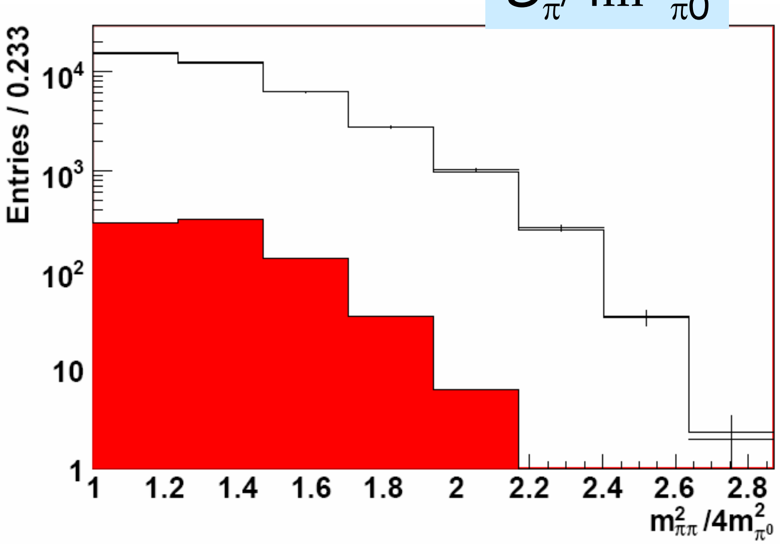


Branching fraction and Form factors measurements:

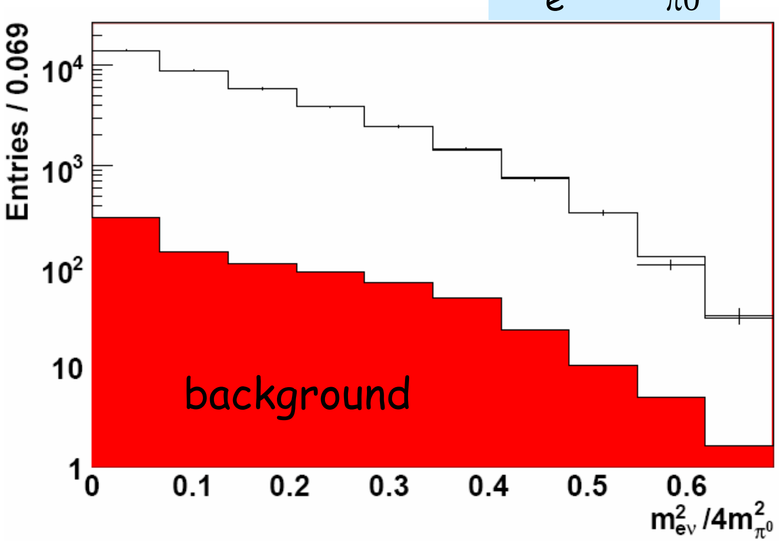
Two identical π^0 → only ONE form factor F_s

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e \left(S_e / 4 m_\pi^{\;2} \right) + ..$$

$S_\pi/4m^2_{\pi 0}$



$S_e/4m^2_{\pi 0}$



Ke4 neutral decays : results

Branching fraction : using 2003 data and ($\pi^\pm\,\pi^0\,\pi^0$) as normalization channel, 9642 signal events (276 ± 94 background events)

$$BR\left(K_{e4}^{00}\right)=\left(2.587\pm0.026_{stat}\pm0.019_{syst}\pm0.029_{ext}\right)\times10^{-5}$$

Form factors (2003+2004 data = ~37000 events):

Se dependence measurement consistent with 0.

$$f_s^{'}/f_s = 0.129\pm0.036_{stat}\pm0.020_{syst}$$

$$f_s^{''}/f_s = -0.040\pm0.034_{stat}\pm0.020_{syst}$$

Ke4 neutral decays : summary

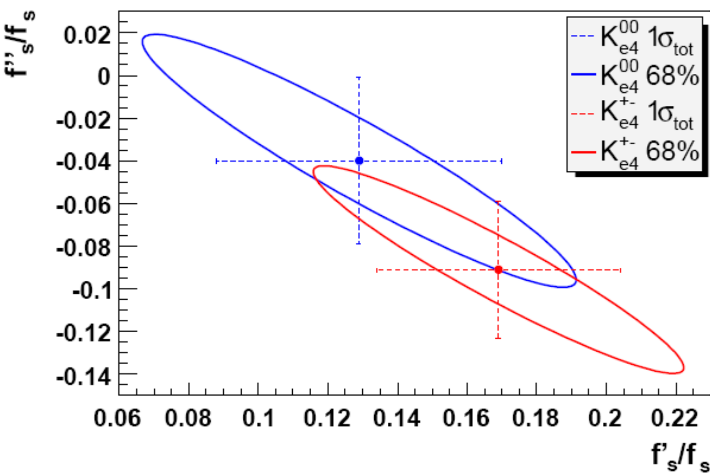
An improved measurement of the BR has been achieved, to be compared with recent published value

KEK-E470 : based on 216 signal events (2.29 ± 0.33) 10^{-5}

This measurement :

$$(2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext})\ 10^{-5}$$

Form factors are measured, consistent with the charged Ke4 measurement



Errors are stat. + syst. assuming same correlation for both.

Using full statistics, the dominant error on BR will be the external error.

Extracting more information from phase shift measurements

	M $\pi\pi$ (GeV/c2)			δ (Radians)	
Bin	min	max	barycenter	value	stat err
1	0.2790	0.2913	0.2860712	0.0438014	0.04396
2	0.2913	0.3005	0.2959924	0.0798576	0.02458
3	0.3005	0.3092	0.3048783	0.0743034	0.02193
4	0.3092	0.3177	0.3134621	0.1303188	0.01815
5	0.3177	0.3263	0.3219863	0.1653183	0.01622
6	0.3263	0.3353	0.3307498	0.1774700	0.01447
7	0.3353	0.3451	0.3401080	0.2135623	0.01462
8	0.3451	0.3570	0.3508481	0.2493961	0.01379
9	0.3570	0.3733	0.3645354	0.2883120	0.01324
10	0.3733	0.4937	0.3900176	0.3378216	0.01240

Results and systematics

- Part of statistics analysed so far
- Systematics uncertainties were investigated and given globally on the single extracted parameter a00 at center of UB.
- Now we can provide systematic uncertainty on individual δ points to allow other approaches.

$$a00\,(UB) = 0.258 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{Theory}$$

Only 30 days from 2003 data were presented

237000 K+

132000 K-

Global approach, revisited and quoted on each point with bin to bin correlations

Here: full UB width

But much more to be done with your help !!

Systematics on individual phase points

Recall : systematics worked out globally for Conference results, components added in quadrature

error	method 1 or 2	accept	Bkg	e ID	Radco	Others params	Total
a0(UB)	0.002	0.002	0.001	0.006	_	0.003	0.007

Systematics for SS123

list of investigated systematics and possible bin to bin correlations

uncorrelated systematics	bin to bin correlated systematics
→ bining choice	Background level
trigger efficiency	S_e dependence
acceptance control	
→ Background shape	
Electron id control	
Radiative corr	
→ Matrix element in bkg	
→ Sign dependent LDA eff.	

Many cross-checks performed by sub-sampling according to various items

- electron impact point at LKr front face (check calorimeter response)
- Kaon charge.
- Achromat and spectrometer polarities
- Time intervals

Correct treatment of correlated errors using a covariance matrix

Systematics: covariance matrix

- To take correctly into account the bin to bin correlations for some systematic errors one should compute an error matrix and use the covariance matrix in the fit :

$$\chi^2 = \sum_{i,j} (x_i - y_i(a)) V_{ij} (x_j - y_j(a))$$

where x is the vector of measurements, y(a) the vector of fitted values for the parameter(s) a and V the covariance matrix of the measurements.

- One build the error matrix :

diagonal terms are the sum of the n uncorrelated errors squared,

$$E_{ii} = \sigma_i 1 \times \sigma_i 1 + \sigma_i 2 \times \sigma_i i2 + \ldots\ldots \sigma_i n \times \sigma_i n$$

off diagonal terms are the sum of the m correlated errors with a correlation coefficient ρ (equal to 1 for full correlation) and no cross correlation between different sources.

$$E_{ij} = (\rho_{ij1} \times \sigma_{i1} \times \sigma_{j1}) + \ldots (\rho_{ijm} \times \sigma_{im} \times \sigma_{jm})$$

- The covariance matrix is the inverse of the error matrix (10 x 10)

If there are no off diagonal terms, the χ2 is the "usual" one

$$\Sigma (x_i - y_i (a))^2 / \sigma_i^2$$

Systematics: covariance matrix

As most systematic uncertainties are bin to bin uncorrelated, the error matrix is almost diagonal (and symmetric), the non-diagonal terms being at least two orders of magnitude lower than the diagonal terms (units are $(mrad)^2$)

bin	1	2	3	4	5	6	7	8	9	10
1	2144.51	7.31	1.19	6.50	4.97	1.59	4.36	0.96	0.88	0.40
2	7.31	715.90	0.95	4.01	3.20	1.10	2.72	0.69	0.60	0.28
3	1.19	0.95	635.21	0.83	0.82	0.36	0.59	0.25	0.19	0.09
4	6.50	4.01	0.83	408.61	2.83	0.97	2.41	0.60	0.53	0.24
5	4.97	3.20	0.82	2.83	378.73	0.85	1.93	0.55	0.46	0.21
6	1.59	1.10	0.36	0.97	0.85	286.86	0.67	0.23	0.18	0.08
7	4.36	2.72	0.59	2.41	1.93	0.67	279.13	0.42	0.37	0.17
8	0.96	0.69	0.25	0.60	0.55	0.23	0.42	276.98	0.12	0.06
9	0.88	0.60	0.19	0.53	0.46	0.18	0.37	0.12	211.15	0.04
10	0.40	0.28	0.09	0.24	0.21	0.08	0.17	0.06	0.04	177.38

Systematics on individual points: Summary

Implementing the systematic errors one at a time, one can compute the covariance matrix and perform the UB center fit to get the corresponding error:

item	a_0^0 value at UB center	Total error	corresponding Systematic error
uncorrelated systematics $\oplus$ stat.			
bining choice	0.25902	0.008376	0.0008
trigger efficiency	0.25902	0.008370	0.0007
acceptance control	0.25847	0.008634	0.0022
Background shape	0.26001	0.008519	0.0017
Electron id control	0.25907	0.008826	0.0029
Radiative corr	0.25892	0.008386	0.0009
Matrix element in bkg	0.25903	0.008341	0.0000
Sign dependent LDA eff.	0.25942	0.008343	0.0002
bin to bin correlated systematics $\oplus$ stat.			
Background level	0.25912	0.008392	0.0009
S_e dependence	0.25906	0.008356	0.0005
statistical error only	0.25903	0.008341	none
statistical error $\oplus$ uncor. syst.	0.25932	0.009397	0.004328
all errors	0.25942	0.009454	0.004450 quad.

Full 2003 statistics : Adding SS0 to SS123

Full Statistics from 2003 is available with same Ke4 selection and background (~0.5%) subtracted, should be ready for KAON2007

Data selected	K+	K-	all
S123	236839	131848	368687 (Conf. result)
SS0	193214	106877	300091
Grand total	430053	238725	668778 = ~80% more

MC generated (for 267 runs = different beam/detector conditions)

accepted events	K+	K-	total
SS123	5.5 Millions	3.0 Millions	8.5 Millions
SS0	4.5 Millions	2.6 Millions	7.1 Millions
Grand total	10.0 Millions	5.6 Millions	15.6 Millions

Ratio K+/K- ~ 1.8 both in Data and MC and run by run
Ratio MC/Data ~ 23. both for K+ and K- and run by run

Summary and outlook

Systematic uncertainties on individual phase points worked out including possible bin to bin correlations. Covariance matrix available for "fitters"

SS0 data will be included soon ,

Statistical error will be reduced : 0.008 →0.006

Systematics to be checked : could decrease as well for the components with statistical origin : 0.005 conservative

Including 2004 data will require more time and efforts ..

Promissing progresses expected with your help in extraction of $\pi\pi$ scattering lengths a00 AND a02