Isospin Violations and Radiative Corrections in $K_{\ell 4}$ Decays

Marc Knecht
Centre de Physique Théorique
CNRS Luminy Case 907
13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

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Theoretical framework :

- Low-energy effective theory, SU(3) framework $(m_u \neq m_d)$

  S. Weinberg (1979)

  J. Gasser and H. Leutwyler (1984, 1985)

- Combined expansion in powers of momenta and α

  R. Urech (1995)

  H. Neufeld and H. Rupertsberger (1995)

- Extension of the framework to the light leptons

  M. Knecht, H. Neufeld, H. Rupertsberger, P. Talavera (2000)

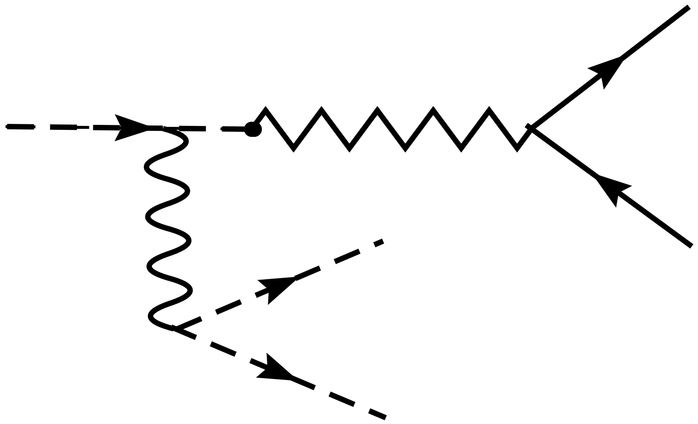
Discuss mainly $K^\pm \to \pi^+\pi^-\ell^\pm\nu_\ell$

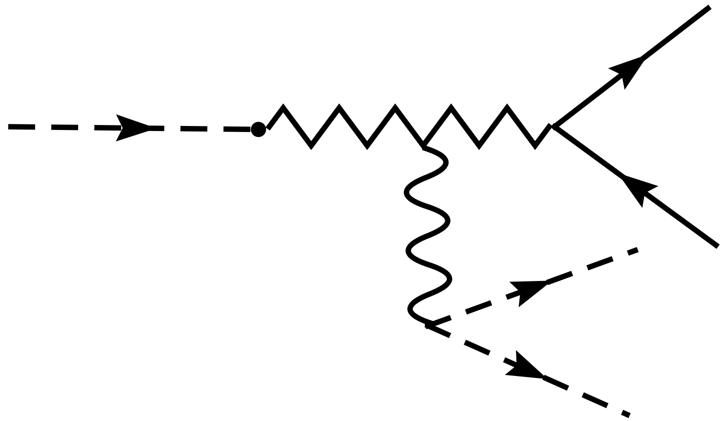
- One advantage: two equal mass pions
$\rightarrow$ invariants have the same expression as in the isospin symmetric case, but with $M_{\pi^\pm}$ instead of M_π

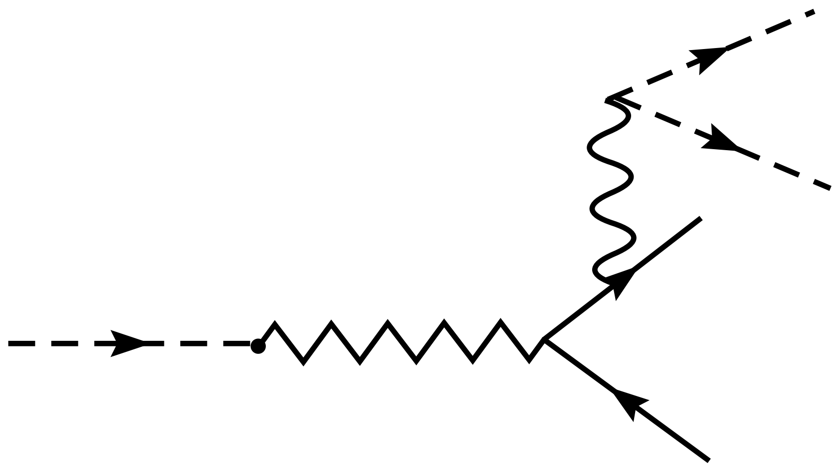
- Big disadvantage: maximal number of charged particles

$\rightarrow$ complicates issue of radiative corrections

Tree level







$$
\begin{aligned}
\mathcal{A}(K^{\pm} \to \pi^+\pi^-\ell^{\pm}\nu_\ell) \; = \\
&= \frac{G_F V_{us}^*}{\sqrt{2}}\, \overline{u}(p_{\nu_\ell})(1+\gamma^5) \times \Bigg\{ \frac{1}{M_{K^\pm}} \Big[(p_1+p_2)^\mu F \\
&+ (p_1-p_2)^\mu G + (p_\ell + p_\nu)^\mu R \\
&+ \frac{i}{M_{K^\pm}^3}\, \epsilon^{\mu\nu\rho\sigma} (p_\ell+p_\nu)_\nu (p_1+p_2)_\rho (p_1-p_2)_\sigma H \Big] \gamma_\mu \\
&+ \frac{1}{2M_{K^\pm}^2}\, [\gamma_\mu, \gamma_\nu]\, p_1^\mu p_2^\nu\, T \Bigg\}\, v(p_\ell)\,.
\end{aligned}
$$

Lowest order, i.e. $\mathcal{O}(p^2e^0)$ and $\mathcal{O}(p^0e^2)$,

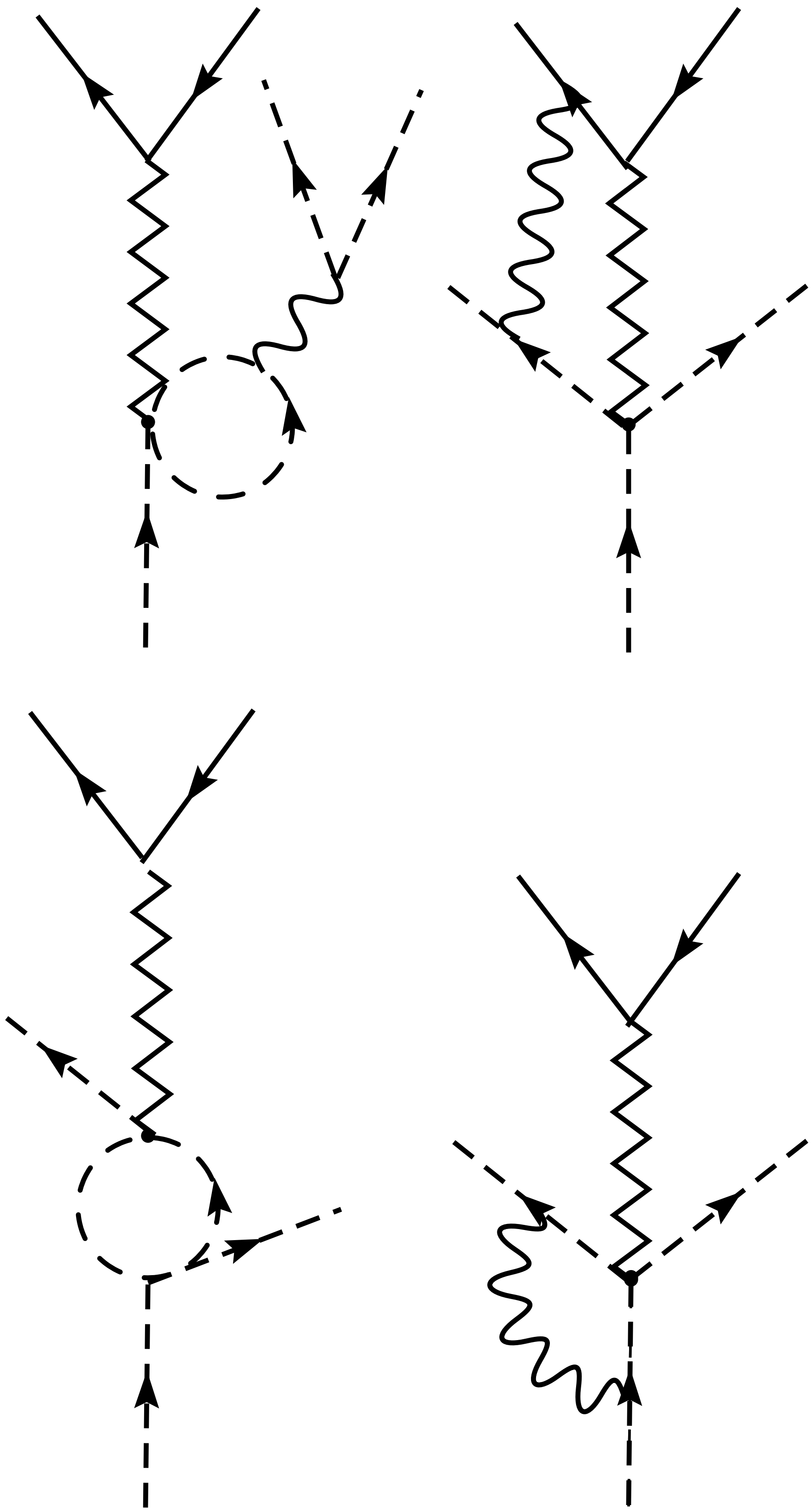
$$
\begin{aligned}
F &= \frac{M_{K^\pm}}{\sqrt{2}F_\pi} \\
G &= \frac{M_{K^\pm}}{\sqrt{2}F_\pi} \\
H &= 0 \\
T &= \frac{M_{K^\pm}}{\sqrt{2}F_\pi}\,\frac{4e^2F_\pi^2}{s_\pi}\,\frac{m_\ell M_{K^\pm}}{M_{K^\pm}^2 - m_\ell^2 - 2k\cdot p_{\nu_\ell}}
\end{aligned}
$$

One-loop order, i.e. $\mathcal{O}(p^4e^0)$, $\mathcal{O}(p^2e^2)$...

- UV divergences are absorbed by the appropriate counterterms

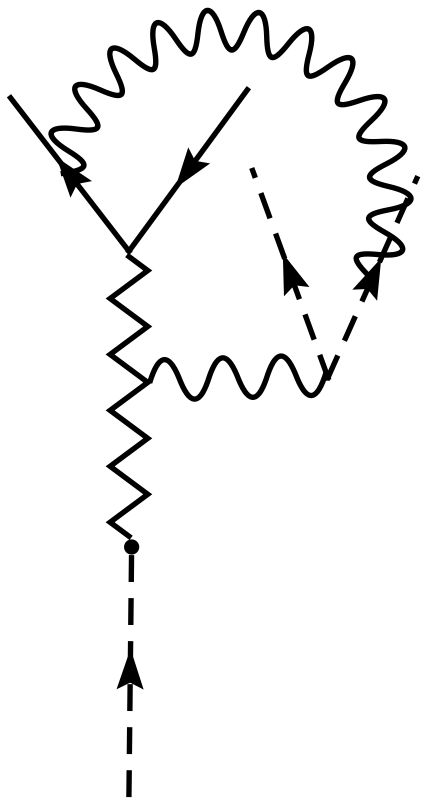
- IR divergences, generated by photon loops, are regulated by a finite photon mass m_γ
$\rightarrow$ see later

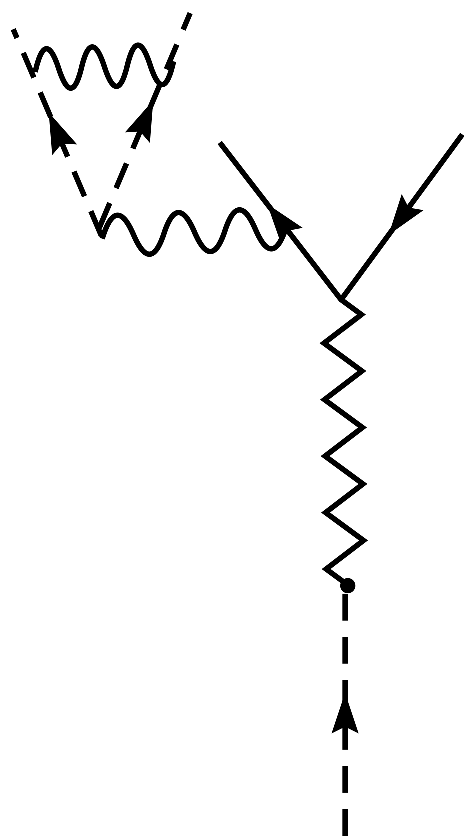
- Expressions for the form factors F and G have been worked out, but remain cumbersome
Cuplov and Nehme, hep-ph/0311274

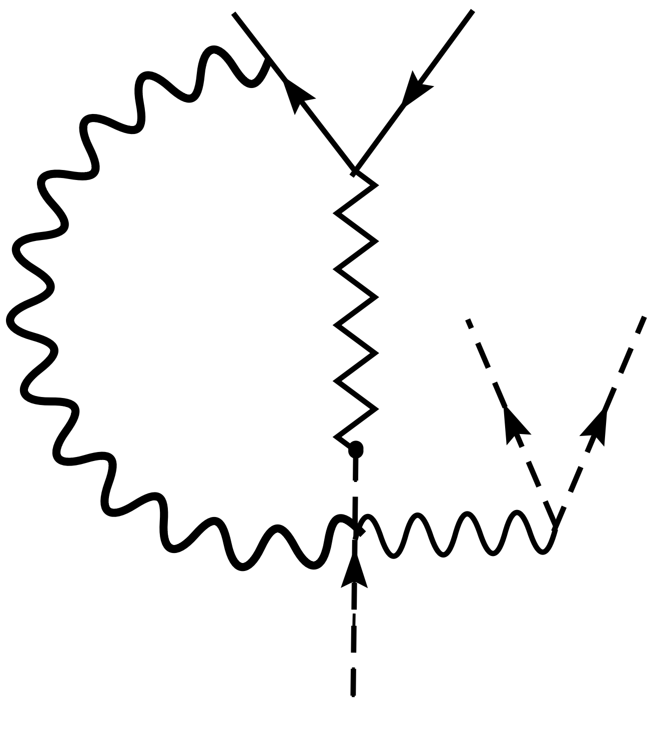


... and $\mathcal{O}(p^0 e^4)$,

- Many diagrams
- Will not be considered







Radiative $K_{\ell 4}$ Decay Amplitudes

$$\mathcal{A}(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell\gamma) = -e\,\frac{G_F V_{us}^*}{\sqrt{2}F_\pi}\varepsilon_\mu^*\mathcal{M}^\mu(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell\gamma)$$

$$\begin{aligned}\mathcal{M}^\mu(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell\gamma) \quad &= \quad \overline{u}(p_\ell)\left(\frac{p_\ell^\mu}{p_\ell\cdot q + m_\gamma^2/2} - \frac{p_+^\mu}{p_+\cdot q + m_\gamma^2/2} + \frac{\gamma^\mu \not{q}}{2p_\ell\cdot q + m_\gamma^2}\right)\\ &\times(2\not{p}_0 - \not{k} + m_\ell)(1-\gamma_5)v(p_{\nu_\ell})\end{aligned}$$

$$\mathcal{A}(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell\gamma) = +e\,\frac{G_F V_{us}^*}{2F_\pi}\varepsilon_\mu^*\mathcal{M}^\mu(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell\gamma)$$

$$\begin{aligned}\mathcal{M}^\mu(K^0 \to \pi^0\pi^0\ell^-\overline{\nu}_\ell\gamma) \quad &= \quad \overline{u}(p_\ell)\left(\frac{p_\ell^\mu}{p_\ell\cdot q + m_\gamma^2/2} - \frac{k^\mu}{p_+\cdot q - m_\gamma^2/2} + \frac{\gamma^\mu \not{q}}{2p_\ell\cdot q + m_\gamma^2}\right)\\ &\times(\not{p}_1 + \not{p}_2 + \frac{2}{3}m_\ell)(1-\gamma_5)v(p_{\nu_\ell})\end{aligned}$$

Radiative $K_{\ell 4}$ Decay Amplitudes

$$\mathcal{A}(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell\gamma) = -e\,\frac{G_F V_{us}^*}{2F_\pi}\varepsilon_\mu^* \mathcal{M}^\mu(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell\gamma)$$

$$\begin{aligned}
\mathcal{M}^\mu(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell\gamma) \quad = \quad & \overline{u}(p_\ell)\Bigg(\frac{p_\ell^\mu}{p_\ell\cdot q + m_\gamma^2/2} - \frac{k^\mu}{p_+\cdot q - m_\gamma^2/2} \\
& + \frac{p_-^\mu}{p_-\cdot q + m_\gamma^2/2} - \frac{p_+^\mu}{p_+\cdot q + m_\gamma^2/2} + \frac{\gamma^\mu\ \not{q}}{2p_\ell\cdot q + m_\gamma^2}\Bigg) \\
& \times(2\ \not{p}_- + \frac{2}{3}m_\ell)(1-\gamma_5)v(p_{\nu_\ell}) \\
+ \quad & \frac{4}{3}\,\overline{u}(p_\ell)\,\frac{p_-^\mu\ \not{q} - (p_-\cdot q)\gamma^\mu}{p_-\cdot q + m_\gamma^2/2}\,(1-\gamma_5)v(p_{\nu_\ell})
\end{aligned}$$

Expand in powers of the photon energy

$$\mathcal{A}(K_{\ell 4\gamma}) = \mathcal{A}_{-1}(K_{\ell 4\gamma}) + \mathcal{A}_{0}(K_{\ell 4\gamma}) + \cdots \tag{1}$$

In order to study the issue of infrared divergences, $\mathcal{A}_{-1}$ is enough (Low approximation)

$$\begin{aligned}
\mathcal{A}_{-1}(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell\gamma) &= e\,\mathcal{A}(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell)\left(\frac{p_\ell\cdot\varepsilon^*}{p_\ell\cdot q + m_\gamma^2/2} - \frac{p_+\cdot\varepsilon^*}{p_+\cdot q + m_\gamma^2/2}\right)\\
\mathcal{A}_{-1}(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell\gamma) &= e\,\mathcal{A}(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell)\left(\frac{p_\ell\cdot\varepsilon^*}{p_\ell\cdot q + m_\gamma^2/2} - \frac{k\cdot\varepsilon^*}{k\cdot q - m_\gamma^2/2}\right)\\
\mathcal{A}_{-1}(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell\gamma) &= e\,\mathcal{A}(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell)\left(\frac{p_\ell\cdot\varepsilon^*}{p_\ell\cdot q + m_\gamma^2/2} - \frac{p_+\cdot\varepsilon^*}{p_+\cdot q + m_\gamma^2/2}\right.\\
&\qquad \left. + \frac{p_-\cdot\varepsilon^*}{p_-\cdot q + m_\gamma^2/2} - \frac{k\cdot\varepsilon^*}{k\cdot q - m_\gamma^2/2}\right)
\end{aligned} \tag{2}$$

Integrate over the three-momenta of the *invisible* particles

$$I_{m,n}(p_1,p_2;P,m_\gamma^2) \equiv \frac{1}{2\pi}\int\frac{d^3\vec{p}_{\nu_\ell}}{|\vec{p}_{\nu_\ell}|}\int\frac{d^3\vec{q}}{\sqrt{\vec{q}^2+m_\gamma^2}} \times\frac{\delta^4(P-p_{\nu_\ell}-q)}{\left[p_1\cdot q+\frac{m_\gamma^2}{2}\right]^m\left[p_2\cdot q+\frac{m_\gamma^2}{2}\right]^n}$$

$$m_\gamma^2 \le x \equiv P^2 \le x_{\text{max}} \qquad (3)$$

$$
\begin{aligned}
d\Gamma_{-1}(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell\gamma) &= \frac{\alpha}{4\pi}\int_{m_\gamma^2}^{x_{\mathsf{max}}} d\Gamma(\overline{K}^0 \to \pi^0\pi^+\ell^-\overline{\nu}_\ell) \\
&\times\Big\{2(p_\ell\cdot p_+)I_{1,1}(p_\ell,p_+;P,m_\gamma^2) \\
&-m_\ell^2 I_{2,0}(p_\ell;P,m_\gamma^2) - M_{\pi^\pm}^2 I_{2,0}(p_+;P,m_\gamma^2)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
d\Gamma_{-1}(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell\gamma) &= \frac{\alpha}{4\pi}\int_{m_\gamma^2}^{x_{\mathsf{max}}} d\Gamma(K^- \to \pi^0\pi^0\ell^-\overline{\nu}_\ell) \\
&\times\Big\{-2(p_\ell\cdot k)I_{1,1}(p_\ell,-k;P,m_\gamma^2) \\
&-m_\ell^2 I_{2,0}(p_\ell;P,m_\gamma^2) - M_{K^\pm}^2 I_{2,0}(-k;P,m_\gamma^2)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
d\Gamma_{-1}(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell\gamma) \quad = \quad & \frac{\alpha}{4\pi}\int_{m_\gamma^2}^{x_{\mathsf{max}}} d\Gamma(K^- \to \pi^+\pi^-\ell^-\overline{\nu}_\ell) \\
& \times\Big\{2(p_\ell\cdot p_+)I_{1,1}(p_\ell,p_+;P,m_\gamma^2) \\
& -2(p_\ell\cdot p_-)I_{1,1}(p_\ell,p_-;P,m_\gamma^2) \\
& +2(p_-\cdot p_+)I_{1,1}(p_-,p_+;P,m_\gamma^2) \\
& -2(p_-\cdot k)I_{1,1}(p_-,-k;P,m_\gamma^2) \\
& +2(p_+\cdot k)I_{1,1}(p_+,-k;P,m_\gamma^2) \\
& -2(p_\ell\cdot k)I_{1,1}(p_\ell,-k;P,m_\gamma^2) \\
& -m_\ell^2 I_{2,0}(p_\ell;P,m_\gamma^2) - M_{K^\pm}^2 I_{2,0}(-k;P,m_\gamma^2) \\
& -M_{\pi^\pm}^2 I_{2,0}(p_+;P,m_\gamma^2) - M_{\pi^\pm}^2 I_{2,0}(p_-;P,m_\gamma^2)\Big\}
\end{aligned}
$$

The integration over x produces $\ln m_\gamma$ terms due to the singular behaviour of the integrands at the lower end of the integration range

$$-\frac{p^2}{4}\int_{m_\gamma^2}^{x_{\text{max}}} I_{2,0}(p;P,m_\gamma^2) \;=\; \ln m_\gamma + \text{finite}$$

$$\frac{p_1\cdot p_2}{2}\int_{m_\gamma^2}^{x_{\text{max}}} I_{1,1}(p_1,p_2;P,m_\gamma^2) \;=$$

$$= \frac{p_1\cdot p_2}{\sqrt{(p_1\cdot p_2)^2-p_1^2p_2^2}}\ln\frac{(p_1\cdot p_2)-\sqrt{(p_1\cdot p_2)^2-p_1^2p_2^2}}{(p_1\cdot p_2)+\sqrt{(p_1\cdot p_2)^2-p_1^2p_2^2}}\ln m_\gamma$$

$+$ finite

Conclusions - Outlook

- K_{e4} is a complex process from the point of view of radiative corrections. The difficulty, however, is technical rather than conceptual.

- Only partial calculations available in the litterature. A user adapted presentation (as e. g. in $K_{\ell 3}$) is missing

- Is the framework adapted ?
$\rightarrow$ no a priori knowledge on $\pi - \pi$ phases